Effects of Problem Context on Strategy Use within Functional Thinking

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Abstract

The effect of problem format on problem solving strategy selection is investigated within the early algebra domain of functional thinking. Functional Thinking is a type of algebraic reasoning appropriate for elementary students, in which a relationship exists between two sets of values. Three function table problems were given to students in grades two through six (N=232) in three different problem contexts. Problem context affected student strategy selection. Presenting the problem with non-indexical X values elicited the most correct strategy use, whereas the format with indexical X values elicited the most naïve and incorrect strategy use. Presenting the problem in a story context did not help correct strategy selection, but it decreased incorrect strategy use. Findings highlight factors influencing strategy selection, and have implication for instructional design and problem solving.

Keywords: Problem Solving; Context; Strategy Use; Mathematics; Functional Thinking; Algebra.

Problem Format and Problem Solving

The format a problem is presented in can affect how well a student understands the underlying concepts and skills the item is tapping (Collins & Ferguson, 1993; Day, 1988; Kirshner, 1989; Zhang, 1997). One problem context is a story context. Teachers and researchers often believe that story problems are harder for students than symbolic problems (Nathan & Koedinger, 2000; Nathan, Long, Alibali, 2002). National assessment data support this notion. Elementary student performance on story problems is generally worse than symbolic problems in the US (Carpenter, Corbitt, Kepner & Reys, 1980; Koba, Carpenter & Swafford, 1989). However, linguistic difficulties seem to account for younger children's poor performance on arithmetic story problems rather than inadequate knowledge of mathematics (Briars & Larkin, 1984; Cummins et al., 1988; de Corte, Verschaffel, & de Win, 1985; Hudson, 1983; Kintsch & Greeno, 1985; Riley et al. 1983).

Once students have proficient linguistic skills, story contexts can have an advantage. In high school students, a verbal advantage of story problems has been found with in algebra and arithmetic. The advantage of story problems was not only a consequence of situated world knowledge facilitating understanding. This advantage was also due to difficulties comprehending the formal symbolic representations of the symbolic problem formats (Koedinger & Nathan, 2004). The effect of problem context was further clarified in college students. A story context was advantageous when the underlying problem was simple, but a symbolic context was best when the problem was complex, presumably because these students had expertise interpreting the symbolic notation (Koedinger, Alibali & Nathan, 2008). Based on these findings, the ideal problem context seems to be dependent on the student's relative familiarity with linguistic and mathematic symbol systems.

When introducing early algebra concepts to elementary school students, mathematics education researchers stress the importance of rich and intuitive background contexts. These are thought to ground students' understanding of the new mathematic concepts they are learning (Carraher, Martinez, & Schliemann, 2008). Story contexts were found to help third grade students solving arithmetic problems over comparable symbolic contexts (Baranes, Perry, & Stigler, 1989). These ideas are in line with learning theories that have emphasized the role of contextual knowledge in supporting the development of symbolic knowledge (e.g., Greeno, Collins, & Resnick, 1996; Vygotsky, 1978).

Problem context can be varied in ways other than adding a story. The presentation of numeric information can be changed in ways which might alter how much attention is given to surface features versus the deep structure of the problem (Bassok, 1996). Having an understanding of the deep structure of a problem is important for fully understanding and correctly solving a problem.

This study investigates the effect of problem context on problem solving strategy within functional thinking, a type of early algebraic reasoning. Functional thinking tasks are appropriate for students that range in age from early elementary, where story context has been shown to hurt, to early middle, where story context has been shown to help. Giving the same task to students in this age range will help elucidate the effect of problem context on problem solving strategy use.

Functional Thinking

Functional Thinking is a type of mathematical thinking which focuses on the relationship between two (or more) varying quantities, specifically the kinds of thinking that lead from specific relationships to generalizations of that relationship across instances (Smith, 2008). The understanding of functions is also one of the core strands of the National Council of Teachers of Mathematics expectations for mathematics curriculum. At the heart of functional thinking is a relationship between two particular quantities that can be described by a *rule of correspondence* (Blanton & Kaput, 2005). This rule of correspondence can be used to find other sets of particular quantities that adhere to the same rule.

Functional Thinking encapsulates some of the most important core components of early algebraic reasoning, such as generalization and covariation, and provides a developmentally appropriate way to scaffold these ways of thought through elementary mathematics education.

X	1	2	3	4	5
Y	5	6	7	8	9

Figure 1: Function Table (Y = X + 4).

Difficulties in Functional Thinking

A critical aspect of functional thinking is understanding the functional relationship between XY pairs. Functional thinking problems can be represented in a *function table* (Figure 1). A function table has an X and a Y column, filled with values that are all related by a function (e.g., Y = X + 4). This is the functional relationship of the table (Carraher et al., 2008). An understanding the functional relationship requires considering the relationship across the columns; between the X and Y values. However, the table can be interpreted another way, by only looking at the relationships within one column, such as between Y_1 and Y_2. This is the recursive relationship within the table (Carraher et al., 2008). Considering this recursive relationship is often temping, particularly when the X values are arranged indexically, with regular intervals, and therefore there are also regular intervals between the Y values. When the problem is presented in this format, to find later Y values in the function table, all one would have to do is extend the pattern within the Y values. However, this relationship is only useful when the X values increase at a constant rate. Additionally, this relationship cannot be efficiently used to predict a Y value for new X value.

Children tend to begin with this recursive strategy, particularly when they are unfamiliar with problems of this type (Carrahar et al., 2008). Broadly speaking, the power of functions is in the functional relationship between X and Y values, so a focus on the recursive relationship is misguided.

Mathematics educators have suggested different problem presentation contexts to help students get out of using the recursive strategy and into using the functional XY strategy. One way is to present the function table with an X axis that has irregular intervals between values (Carraher & Earnest, 2003; Warren & Cooper, 2005), or even clearly defined visual breaks in the table structure itself (Carraher et al., 2008; Schliemann, Carraher, & Brizuela, 2001). These break up the regular pattern in the Y values, thus discouraging children's strategy of simply looking to only the pattern in the Y values to determine the missing Y values later in the table.

Another way this can be overcome is to present the function problem with a story context, so the student can have an intuitive understanding of the underlying functional relationship. A story context can help ensure that students are considering the relationship between multiple input and output values (Schliemann et al., 2001). In this way, students are less likely to utilize a shallow recursive strategy. These instructional techniques help guide students away from the initial recursive strategy and into the correct functional strategy.

There is much writing as to which problem contexts are best for learning, but no systematic investigation for elementary level functional thinking problems. This study investigates the effect of problem context on strategy use within function table problems.

X	Y
2	6
3	7
4	8
5	9
6	
14	
	25
41	

X	Y
2	6
4	8
5	9
7	11
8	
14	
	25
41	

Cost of Present	Cost of Present with Gift Wrapping
2	6
3	7
4	8
5	9
6	
14	
	25
41	

Story Context: At a gift shop, you can pay extra to have your present gift-wrapped, as shown in the table below. What is the *total cost of the present with gift-wrapping* if the *cost of the present* is $6? $14? What about $41? If the *total cost of a present with gift-wrapping* is $36, what was the *cost of the present* itself?

Figure 2: Function Table Formats. Indexical, Non-Indexical, and Story Context.

Method

An assessment on functional thinking was given to students in grades two through six. Three function table items were included, which asked the students to fill in missing Y values in a function table, and to find the rule of correspondence. The underlying functions for these items were additive (Y = X + 4), multiplicative (Y = 3X), and a combination (Y = 3X + 2). These three items were presented in three contexts: a function table with indexically increasing X values and no story context (*indexical*), a function table with non-indexically increasing X values and no story context (*non-indexical*), and a story problems with indexically increasing X values (*story*) (see Figure 2). The items in these three conditions were kept as similar as possible, with the only differences being the factors that we were manipulating. The story contexts were about the cost of having a present gift-wrapped, saving money for a bicycle, and how many people could be seating at different arrangements of dinner tables. These story contexts were adapted from instructional materials created by math education researchers (e.g. Schliemann et al., 2001). The rule was not articulated in the story context and had to be deduced from the function table values. Each individual assessment contained the additive, multiplicative, and combination function table problems in the same context condition. All problems on a given assessment were in the same format, therefore there were three versions of the assessment; indexical, non-indexical, and story.

The assessments were randomly distributed to 232 2nd through 6th grade students in a middle class suburban elementary and middle school in the southeastern United States. The general instructions for the assessment were read aloud to the second grade students, but they read the individual problems themselves.

Coding The students' work was coded for strategy use. The student's strategy was determined from the values they wrote in the function tables. Students' strategy use was coded as *correct* if they used the correct functional strategy, *recursive* if they used a recursive strategy, and *other*. If the student gave the correct entries, regardless of a correctly written rule of correspondence, or gave an incorrect entry for one blank, but gave a correct rule of correspondence, they were coded as *correct*. Students were given a *recursive* code if they had filled in the table by looking at the pattern in one column, instead of the relationship between the two columns. Students often used *other* strategies, such as an incorrect functional strategy, a mix of a functional and recursive strategy, an indiscernible strategy, or if the student left the table blank. There were no systematic differences in other strategy use of these types between conditions, and so they were collapsed in all further analyses. See Table 1 for a breakdown of strategy use by condition. Only correct and recursive strategy use was considered in this analysis.

Strategy	Description	Sample Student Response	Frequency		
			Index	Story	Non Index
Correct	Used correct functional rule to fill in table	**Y = 3X** X: 2 3 4 5 6 12 52 Y: 6 9 12 15 **18 36 156**	39.1%	39.4%	48.3%
Recursive	Filled in table following Y pattern, instead of between X and Y	**Y2 = Y1 + 3** X: 2 3 4 5 6 12 52 Y: 6 9 12 15 **18 21 24**	16.4%	9.1%	6.25%
Other	Incorrect Functional, Mixed Functional and Recursive, Unclear, and Blank		44.5%	51.5%	45.5%

Coded as strategy even if one entry in the table was incorrect or blank

Table 1: Strategy Use Percentages by Condition

Results

We compared the effect of problem context (indexical, non-indexical, and story) on strategy use (correct or recursive). There was an overall effect of problem format on both correct and recursive strategy use.

Correct Strategy Use

The correct strategy was utilized the most overall, with it being used 39% of the time in both the indexical and story context, and 48% of the time in the non-indexical context (See Figure 3). The effect of problem context on correct strategy use was evaluated through a series of ANCOVAs with correct strategy use as a dependent variable, condition and grade as between subjects factors and a grade by condition interaction term. Grade was treated as a continuous variable. The initial model tested for a grade by condition interaction, which was not significant, and therefore the interaction term was dropped from all further analyses. Problem context had a significant effect on correct strategy use, $F(2, 225) = 3.23$, $p = .042$, $\eta^2 = .028$. A post hoc analysis of correct strategy use revealed that differences between conditions were significant when comparing the *non-indexical* and *story problem* contexts, $F(1,151) = 5.74$, $p=.018$, $\eta^2 = .037$. The difference between the *indexical* and *non-indexical* was marginal, $F(1,149) = 2.778$, $p=.098$, $\eta^2 = .018$. There was no difference in correct strategy use in the *indexical* and *story* contexts $F(1,146) = .523$, $p = .47$, $\eta^2 = .004$. This pattern of results was the same when students were split into younger (2nd and 3rd) and older (4th through 6th) groups, showing that this effect was not dependent on grade. Average accuracy performance was similar within these groupings, and so were collapsed for summative analyses. The younger students used the correct strategy in the *non-indexical* condition the most (29.1%), and less in the *indexical* and *story* contexts (15.5% and 11.6%). The older students used it 66.7% in the *non-indexical* context, and 55.8% and 61.2% in the *indexical* and *story* contexts. Overall, the *non-indexical* context was the most conducive to the correct problem solving strategy, and there was no difference in strategy use in the *indexical* and *story* contexts.

Recursive Strategy Use

The recursive strategy was utilized less often, with it being used 16% of the time in the *indexical* context, 6% of the time in the *non-indexical* context, and 9% of the time in the *story* context. Problem context had a significant effect on recursive strategy use $F(2, 225) = 3.49$, $p=.032$, $\eta^2 = .03$. There was a significant difference in strategy use between the *indexical* and *non-indexical* contexts $F(1,149) = 6.217$, $p=.014$, $\eta^2 = .04$. There was no significant difference when directly contrasting the other conditions (*story* vs. *non-indexical*, $F(1,151) = 1.003$, $p=.318$, $\eta^2 = .007$; *story* vs. *indexical*, $F(1,146) = 2.28$, $p=.133$, $\eta^2 = .015$). Again, this pattern of results was the same when students were split into younger (2nd and 3rd) and older (4th through 6th) groups. The younger students used the recursive strategy in the *indexical* condition the most (28.2%), and less in the *non-indexical* and *story* contexts (13.6% and 13.6%). The older students used it 6.2% in the *indexical* context, and 2.3% and 5.4% in the *non-indexical* and *story* contexts. This effect was not dependent on grade. Interestingly, there was a trend towards a stronger effect of problem context on recursive strategy use when the type of underlying function (i.e., multiplicative) was difficult for the student. The *indexical* context elicited the most recursive strategy use, and there was no difference in strategy use in the *non-indexical* and *story* contexts.

Discussion

Problem context had an effect on problem solving strategy use. Particularly, the *non-indexical* context encouraged the use of the correct strategy relative to other formats, and the *indexical* context encouraged the use of the recursive strategy relative to other formats. Interestingly, the *story* context discouraged use of the recursive strategy, but it did not encourage use of the correct strategy.

These findings have direct implications for the teaching and learning of function tables. In pedagogical contexts, function table problems should be presented with non-indexical X values to facilitate student understanding.

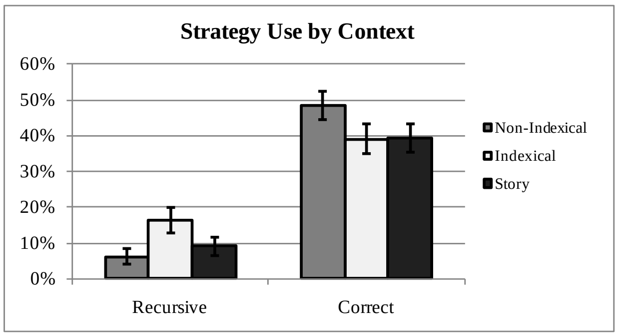

Figure 3: Problem Solving Strategy Use by Problem Context.

The indexicality of the X values had a large effect on student strategy use. Particularly, the *indexical* context encouraged the use of the naïve and incorrect recursive strategy. This could be the case because the students may have utilized the surface feature of the constant pattern in the Y values, and found it sufficient to determine the missing values. Specific aspects of content, context, and phrasing of a problem often play a crucial role in helping people determine the structure of a problem. Because of this, different structures may be abstracted from formally isomorphic problems that have different surface features (Bassok, 1996). The differences in surface features between function tables with indexical and non-indexical X values seemed to have been enough to invoke different structural interpretations in students. By arranging surface features of a problem, the learner's attention can be directed in more or less efficient manner to the underlying structure.

The story context did reduce the use of the naïve recursive strategy, but it did not support use of the correct strategy. Previous research suggests that the benefits of story contexts are dependent on the learner's relative familiarity with the linguistic and mathematic symbol systems (Koedinger, Alibali & Nathan, 2008; Rittle-Johnson & Koedinger, 2005). Our population included a range of students whose reading ability varied from novice to proficient, allowing us to address the effect of story context on students with different reading skill. The second and third grade students read the story context to themselves, yet the pattern of results between conditions was the same as those of the older students. This suggests that reading difficulties were not an issue, and that there was no verbal disadvantage for younger students. Standardized state test data was available for a subset of the 3rd through 6th grade students, and performance on our whole functional thinking assessment did not highly correlated with reading scores ($r(89) = .613$, $p < .01$). The story context seemed to reduce the tendency to focus on the Y_1Y_2 recursive relationship. This may be because the familiar and semantic information contained in the story helped form the students' understanding of the underlying problem structure. However, this story context was not enough to encourage correct strategy use, by considering the XY relationship. This effect of story context may be different from previous research findings, as the domain of functional thinking does not, at this elementary level, involve any mathematic symbolic notation. The problems only contain whole numbers, and the new concept is the focus on the XY relationship. As such, story contexts might not have as great a benefit as they do in arithmetic and algebra.

In this study, we wanted to isolate the effects of indexicality of the X values and a story context. How the two problem presentation features would interact was an open question. Given the results of this study, it is clear that future investigations should include story contexts with non-indexical X values. Perhaps the combination of both the real world context and numeric values without tempting surface patterns will be the most powerful in facilitating correct functional strategy use.

This study shows that seemingly small changes in problem context can affect the strategies a learner uses to solve a problem.

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References

Baranes, R., Perry, M., & Stigler, J. W. (1989). Activation of real-world knowledge in the solution of word problems. *Cognition and Instruction, 6*, 287–318.

Bassok, M. (1996). Using Content to Interpret Structure: Effects on Analogical Transfer. *Current Diections in Psychological Science, 5*(2), 54-58.

Blanton, M., & Kaput, J.J. (2005). Characterizing a classroom practice that promotes algebraic reasoning. *Journal for Research in Mathematics Education, 36*(5), 412-446.

Briars, D. J., & Larkin, J. H. (1984). An integrated model of skill in solving elementary word problems. *Cognition and Instruction, 1*, 245–296.

Carpenter, T. P., Corbitt, M. K., Kepner, H. S., & Reys, R. E. (1980). Solving verbal problems: Results and implications from national assessment. *Arithmetic Teacher, 28*(1), 8–12.

Carraher, D., Martinez, M., & Schliemann, A. (2008). Early algebra and mathematical generalization. *ZDM Mathematics Education*, 403-422.

Carraher, D.W., & Earnest, D. (2003). Guess my rule revisited. *Proceedings of the 27th International Conference for the Psychology of Mathematics Education*, Honolulu, HI.

Collins, A., & Ferguson, W. (1993). Epistemic forms and epistemic games: Structures and strategies to guide inquiry. *Educational Psychologist, 28*, 25–42.

Cummins, D., Kintsch, W., Reusser, K., & Weimer, R. (1988). The role of understanding in solving word problems. *Cognitive Psychology, 20,* 405–438.

Day, R.S. (1988). Alternative representations. In G.H. Bower (Ed.), *The psychology of learning and motivation* (Vol. 22). San Diego, CA: Academic Press.

de Corte, E., Verschaffel, L., & deWin, L. (1985). Influence of rewording verbal problems on children's problem representations and solutions. *Journal of Educational Psychology, 77,* 460–470.

Greeno, J.G., Collins, A.M., & Resnick, L.B. (1996). Cognition and learning. In D. Berliner & R. Calfee (Eds.), *Handbook of educational psychology*. New York: Macmillan.

Hudson, T. (1983). Correspondences and numerical differences between disjoint sets. *Child Development, 54,* 84–90.

Kirshner, D. (1989). The visual syntax of algebra. *Journal for Research in Mathematics Education, 20*, 274–287.

Koedinger, K.R., & Nathan, M.J. (2004). The Real Story Behind Story Problems: Effects of Representations on Quantitative Reasoning. *Journal of the Learning Sciences, 13*(2), 129-164.

Koedinger, K.R., Alibali, M. W., & Nathan, M.J. (2008). Trade-Offs Between Grounded and Abstract Representations: Evidence from Algebra Problem Solving. *Cognitive Science, 32*(2), 366-397.

Kouba, V.L., Carpenter, T.P., & Swafford, J.O. (1989). Number and operations. In M.M. Lindquist (Ed.), *Results from the fourth mathematics assessment of the national assessment of educational progress* (pp. 64–93). Reston, VA: National Council of Teachers of Mathematics.

Nathan, M.J., & Koedinger, K.R. (2000). Teachers' and researchers' beliefs of early algebra development. *Journal of Mathematics Education Research, 31*(2), 168–190.

Nathan, M.J., Long, S.D., & Alibali, M.W. (2002). Symbol precedence in mathematics textbooks: A corpus analysis. *Discourse Processes, 33,* 1–21.

Riley, M.S., Greeno, J.G., & Heller, J.J. (1983). Development of children's problem-solving ability in arithmetic. In H. Ginsburg (Ed.), *The development of mathematical thinking*. New York: Academic.

Rittle-Johnson, B., & Koedinger, K.R. (2005). Designing Knowledge Scaffolds to Support Mathematical Problem Solving. *Cognition and Instruction, 23*(3), 313-349.

Schliemann, A.D., Carraher, D.W., & Brizuela, B.M. (2001). When tables become function tables. *Proceedings of the XXV Conference of the International Group for the Psychology of Mathematics Education*, Utrecht, The Netherlands.

Smith, E. (2008). Representational Thinking as a Framework for Introducing Functions in the Elementary Curriculum. *Algebra in the Early Grades*. J. J. Kaput, Carraher, D. W., Blanton, M. L. New York, NY, Lawrence Erlbaum Associates.

Vygotsky, L.S. (1978). *Mind in society: The development of higher psychological processes.* Cambridge, MA: Harvard University Press.

Warren, E., & Cooper, T. (2005). Introducing Functional Thinking in Year 2: a case study of early algebra teaching. *Contemporary Issues in Early Childhood 6*, 150-162.

Zhang, J. (1997). The nature of external representations in problem solving. *Cognitive Science, 21*, 179–217.