Spatial Reasoning as Verbal Reasoning

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Abstract

We introduce an approach for how spatial reasoning can be conceived as verbal reasoning. We describe a theory of how humans construct a mental representation given one-dimensional spatial relations. In this construction process objects are inserted in a dynamic structure called a "queue" which provides an implicit direction. The spatial interpretation of this direction can be chosen freely. This implies that choices in the process of constructing a mental representation influence the result of deductive spatial reasoning. To derive the precise rules for the construction process we employ the assumption that humans try to minimize their cognitive effort, and a cost measure is introduced to judge the efficiency of the construction process. From this we deduce how the queue should be constructed. We discuss empirical evidence for this approach as well as a computational implementation of the construction process.

Keywords: Verbal Reasoning; Spatial Reasoning; Mental Models; Cost Function; Computational Framework

Introduction

One dimensional spatial relations like "right of", "left of", "in front", "behind", "north of" have in common that they are transitive and, thereby, allow us to create a linear order between objects linked by one of these relations. Let us demonstrate this by an example. Consider the following two sentences, also called premises.

1. The apple is to the left of the mango.
2. The mango is to the left of the kiwi.

These premises allow us to create a linear order of the objects named in the premises, apple–mango–kiwi. This order enables us to draw conclusions about information not directly given in the premises: we can infer that the apple is to the left of the kiwi. The ability to infer information about relations between objects not explicitly yielded by the premises is the subject of theories about relational reasoning (cf. Johnson-Laird & Byrne,1991; chapter 5). The bases of such inferences are internal representations that reflect information conveyed verbally by the premises. There are several theories on how this is accomplished. Syntactic-based approaches (Braine & O'Brien, 1998; Rips, 1994; Hagert, 1984; Henst & Schaeken, 2005) suggest that the reasoning process is based on operations similar to the syntactic rules of formal logic. A set of rules is applied to draw inferences from given premises in order to derive new information implicitly provided by the premises. Model-based approaches, such as the mental model theory (MMT) on the other hand, suggest that reasoners infer new information by inspecting a mental model, representing the "state of affairs", described by the premises (Johnson-Laird & Byrne, 1991).

Polk and Newell (1995), however, point out that the deduction process does not necessarily require deduction-specific mechanisms to operate on internal representations. Especially in reasoners that are not specifically trained on deductive reasoning more general cognitive mechanisms might guide the reasoning process. They introduced a third approach, called **verbal reasoning**, that assumes the cognitive processes in deductive reasoning to be based upon the same processes as language comprehension and generation. Verbal reasoning describes reasoning as transformation of verbal information provided by the premises of an inference problem. Linguistic skills operate in order to encode and re-encode a reasoning problem until the conclusion becomes obvious or until the reasoner gives up. Polk and Newell (1995) hypothesize that when task-relevant information is provided verbally, the crucial role in reasoning is played by the verbal processes of encoding and re-encoding accordingly and that inferences follow comparatively easily from the encoded information.

In the following, we sketch how spatial reasoning can be conceived in Polk and Newell's framework of verbal reasoning, which covers reasoning about relations. In particular, we propose new theoretical assumptions for the special case of reasoning with spatial relations. The key assumption is that the process of constructing a mental representation – a mental model – from the premises influences deductive spatial reasoning. This implies that the process of encoding the spatial information is critical for the result of the spatial reasoning process. We discuss empirical evidence as well as a computational implementation of the process.

A cognitive model

We are proposing a theory on how humans create a mental model from a set of spatial relations. The theory is based on the idea of cognitive efficiency, that is humans try to minimize their cognitive effort, therefore a cost measure is introduced to judge the efficiency. From this we derive how a mental model should be constructed. This mental model can then be used to reason about spatial relations and its properties imply consequences for the reasoning process.

Basic assumption for the cognitive model

Since we consider arbitrary one-dimensional relations as basis for the model we assume that models consist of a "queue" of objects and an interpretation what this queue represents. The queue describes in which order the objects are aligned but what this order represents depends on the relation that is considered. So while the order is implicit the interpretation of the order is not. The queue is constructed by forming links between objects. The links signify which objects follow each other in that ordered arrangement. These links between the objects are one directional which means that when inspecting the queue we can move from one object to the next object in the order but not to the preceding object. To access the queue one needs to access the first element of the queue. Therefore the beginning of the queue is marked by a start pointer.

The queue can be accessed from this starting point which is directed at the first object. From there all other objects in the mental model can be reached by following the links between objects.

This amounts to the following assumptions about the queue:

1. There exist a starting point or first object.
2. Each object is linked to the next object in the linear order. Only the last object is not linked to other objects.
3. While this structure has an implicit direction, the interpretation of this direction depends on the context.

Mental Cost

We now introduce a cost measure that allows us to judge how to create the queue efficiently. The main assumption is that an existing link should not be broken if that is avoidable and as few new links as possible should be formed to minimize cognitive work.

So a cost efficient model is one that can be built by a minimal number of broken links. Since in the end of the construction process the complete mental model is supposed to have as many links as objects costs can only be reduced by altering as few links as possible during the construction process. Therefore it is most cost efficient if we can insert new objects creating just one new link and not changing any existing links. The only way to accomplish this is by attaching them at the very end, following the last object in the queue. So if an object can be inserted at the very end of the queue it should be inserted there.

The starting point is also considered a link. This is due to the fact that one has to know how the queue starts in order to access it. Therefore knowing which object is the first constitutes a link, connecting the start of the queue to that object.

Moving through the queue on the other hand takes very little cognitive effort as long as we move in the implicit direction of the queue. Due to the fact that the links only have one direction moving in the opposite direction through the queue is impossible.

Construction of mental models from spatial information

The question now is how a mental model is constructed from the premises of a reasoning problem. How are objects inserted in the queue and where does the cost measure come into play?

In this process the first premise that is considered has a special function and dominating effect on the construction of the rest of the arrangement. We consider the first premise independently of the following premises and postulate the following two rules for the construction process.

1^{fp} First object inserted in the queue is the starting point of the queue.

2^{fp} The second object is linked to the first object. The relation between the first and the second object thereby creates the interpretation of the link and the implicit direction of all the following objects in the queue.

If we know, for example, that the second inserted object is supposed to be to the right of the first (starting) object, then the link is interpreted as "to the right".

When we look again at our example from the introduction this gives us two options for the first premise: "The apple is to the left of the mango." We can choose the apple as the starting point (marked by the asterisk) and insert the mango second:

$$\text{apple}^* \rightarrow \text{mango}$$

The implicit direction of the queue is interpreted as moving from the leftmost object to the right. However, if we use the mango as a starting point (marked by the asterisk) inserting the apple second we get:

$$\text{apple} \leftarrow \text{mango}^*$$

In this case the implicit direction of the queue is interpreted as moving from the rightmost object to the left. So even though the premise describes only one arrangement of fruits there are two options for representing this arrangement in our queue.

Once the interpretation of the implicit direction of the queue is fixed by inserting the second object the rest of the objects are inserted according to this interpretation. This amounts to the following options for inserting objects in an existing queue from the second premise on:

1. The first object of the premise has to be found in the queue.

2.(a) If the new object is to be placed behind this object (with regard to the implicit direction of the queue) it can be either inserted into the queue directly behind the object or at any point further to the end of the queue.

(b) If the new object is to be placed in front of the object (with regard to the implicit direction of the queue) it can be either inserted into the queue directly in front of the object or at any point further to the beginning of the queue.

The question is which of these choices is more cost efficient. As a cost measure we use primarily the number of links that need to be formed. If this does not show any difference between the options the required movement through the queue is used as an secondary cost measure.

Let us first look at the costs resulting from inserting a new object into the queue between two objects that are linked. To insert a new object between two existing objects in the queue the first object, that was linked to the second object before, is now linked to the new object. The new object is linked to the second object. This requires forming two new links. If the object is inserted at the beginning of the queue the starting point needs to be redefined which we will consider as creating a new link.

Using this information we will now judge the cost created by the insertion options described in 2.(a) and (b). Let us first look at option (a): If the object is inserted between two objects of the queue two new links need to be formed. If the object is inserted at the end of the queue, only one new link needs to be formed. So in case (a) it is most cost efficient to insert the object at the very end of the queue. Now we consider (b): The new object can only be inserted between two objects or at the starting point of the queue. Since we consider the starting point a link to the beginning of the queue both options require two new links to be formed. So it is the most cost efficient not to move around the queue but to insert the object directly in front of the found object. Using this analysis we postulate the following rules:

1^{ins} If the new object is to be placed behind an object of the queue it will be inserted *at the end* of the queue.

2^{ins} If the new object is to be placed in front of an object of the queue it will be inserted into the queue *directly in front* of this object.

If we apply these rules to the second premise of the first example we create one of the following two models depending on the direction of the queue.

$$\text{apple}^* \rightarrow \text{mango} \rightarrow \text{kiwi} \qquad (1)$$

$$\text{apple} \leftarrow \text{mango} \leftarrow \text{kiwi}^* \qquad (2)$$

While the results look similar, the costs for building these models differ. In case (1) we were able to use rule 1^{ins}, creating only one more link. In case (2) however, we needed to redefine the starting point. This resulted in creating two new links. So the cognitive costs for building the first model are lower.

Let us look at another example that is not quite as simple:

1. The apple is to the left of the mango.

2. The apple is to the left of the kiwi.

Here the premises describe an indeterminate order: there are two possible orders of these three fruits: apple–mango–kiwi and apple–kiwi–mango. So the question is, whether one of these orders is preferred over the other? Knauff, Rauh, and Schlieder (1995); Rauh et al. (2005); Jahn, Knauff, and Johnson-Laird (2007) have empirically shown that such preferences exist in human reasoners.

Since the first premise is the same as in the example with the determinate order we receive the same two options for models when applying the rules for the first premise. If we apply the rules of insertion to the second premise we get one of the following models.

$$\text{apple}^* \rightarrow \text{mango} \rightarrow \text{kiwi} \qquad (3)$$

$$\text{apple} \leftarrow \text{kiwi} \leftarrow \text{mango}^* \qquad (4)$$

Here we see a difference between the two models depending on the implicit direction of the queue. This is due to the fact that the arrangement is indeterminate and because the two queues have opposite interpretations of the implicit direction different rules are applied to form the queues. There is also a difference in the cost for building these models. In (3) we were able to apply rule 1^{ins}, again creating only one new link. In (4) we needed to apply rule 2^{ins}, redefining the starting point, creating two new links. So the cognitive costs for creating the last model (4) are higher than the ones for creating model (3).

Once a model has been constructed it can be used to make inferences. If we build the model

$$\text{apple}^* \rightarrow \text{mango} \rightarrow \text{kiwi}$$

from the premises of the first example we can answer the question "Is the apple to the left of the kiwi?" by finding the apple in the queue and then moving further down the queue till we find the kiwi.

Predictions based on the construction process:

From the model we can derive several behavioural predictions:

- If the model is indeterminate (allowing more than one model) the direction of the queue influences which model will be built.

- It should be easier to infer information that can be obtained following the implicit direction of the queue than infer information that require to go against that direction.

- The same mechanism is used for all one dimensional spatial relations, not just in the left/right direction.

Computational implementation and computational complexity estimation

The model construction process can be easily implemented as a computer model using the data structure linked list, consisting of nodes containing data and a pointer to the next node in the list. There is also a start pointer pointing at the first node of the list. If we compare that to our mental model the pointers from one node in the list to the next represent the link between the objects. The data represent the objects. It is therefore easy to model a queue such as the one we proposed in a computer program.

Computer science provides standard rules for analysing the efficiency of algorithms. However, the traditional cost analysis of the algorithm used to insert objects into a linked list provides different results than the above cost analysis. This is because in computational complexity theory every operation has the same weight. There are no operations that are harder or easier to perform than other operations.

Let us look again at the possibilities for insertion in an existing queue from above, 2(a) and (b). Which of these options is the most cost efficient? When inserting a node behind a node of the list as in 2(a), and we insert it directly behind the found node, we have the cost of assigning one pointer and reassigning another (if not the end of the list). If we move further down the list, the costs of moving through the list have to be added to the costs of assigning pointers and moving one node down the list costs as much as assigning one pointer. So inserting a node between two nodes further down the list is always more expensive than inserting it right behind the found node. Attaching the node to the end of the list is not a good idea either: if the end of the list is more than one node away, the cost of moving through the list and assigning the pointer will be higher than the cost of just inserting the new node right behind the found node. And since there is no way of knowing how far away the last node is, the cost efficient solution is to insert the new node right behind the found node.

When inserting a node in front of a found node of the list as in 2(b), the same costs result for inserting the new node right in front of the node and for inserting it at the beginning of the list as the starting pointer of the list can always be accessed at no extra cost. In both cases one pointer needs to be assigned and one pointer needs to be redirected. If it is inserted at any other point of the list the costs are higher since we first have to move to that point from the beginning of the list. So in this cost analysis it would be most efficient to insert the object either at the beginning of the list or directly in front of the found object.

Based on this analysis we derive alternative rules for inserting nodes into a list:

1^{alt} If the new node has to be placed behind a node of the list it should be inserted into the list *directly behind* the node.

2^{alt} If the new node has to be placed in front of a node of the list it should be inserted into the list *either directly in front* of this node or at the *very beginning* of the list.

If we apply these rules to the second premise of the second example we receive the following models.

$$\text{apple}^* \rightarrow \text{kiwi} \rightarrow \text{mango} \qquad (5)$$

$$\text{apple} \leftarrow \text{mango} \leftarrow \text{kiwi}^* \text{ OR apple} \leftarrow \text{kiwi} \leftarrow \text{mango}^* \quad (6)$$

The model (5) was built using rule 1^{alt}, the models in (6) are the two options following from rule 2^{alt}. The insertion of the last object has the same computational cost in all three of these models.

The models also show that rules based on a classic computational cost measure produce different results than our rules based on a cognitive cost measure. Model (5) differs from model (3) above and only one of the models of (6) is similar to the model (4). Of course this does not mean that a computational model would have to follow the alternative rules. It can also be implemented using the insertion rules that resulted from the cognitive model.

So one of the questions is whether it is justified to assume that forming a link is more cost intensive than moving through the queue. If not, the traditional computational complexity measure might provide better predictions than our model.

Empirical Evidence

We report an experiment that shows that rules derived from our cognitive cost measure predict human behaviour better than the rules derived from the traditional computer science cost measure. In this experiment we investigated what kind of mental model participants construct when they are faced with indeterminate problems that allowed more than one model to be constructed.

Participants, Materials, Procedure, and Design

Thirty-five participants (3 male; age: $M = 22.4$; $SD = 3.2$) from the University of Giessen had to solve sixteen determinate (like in example 1) and sixteen indeterminate problems (like in example 2). The three-term problems had two premises each and we used only the relation “left of”. The problems were presented to the participants in a random order on a computer screen. Each premise was presented sequentially (in a self-paced manner). After having read the premises a conclusion was presented and the participants were asked if this conclusion was correct or not. For determinate problems the conclusion was either true or false. For indeterminate problems we used two different types of conclusions which could either hold in a model constructed according to rule 1^{ins} or to rule 1^{alt}.

Results and Discussion

Separate ANOVAs for the percentage of correct responses and reaction times of correct responses with the within-subject factor conclusion acceptance (hits, correct rejections) and insertion principle (indeterminate/rule1^{ins} vs. indeterminate/rule1^{alt}) were calculated, respectively. Level of significance was 5%.

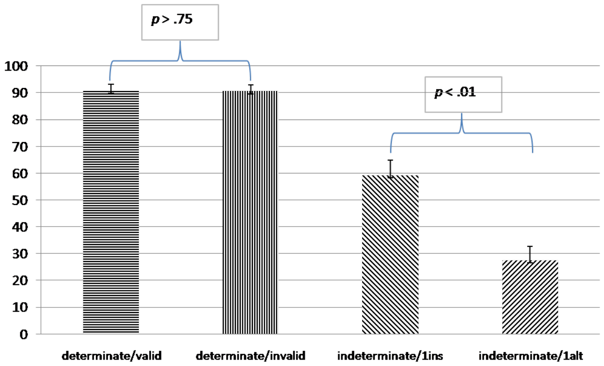

Figure 1: The left two bars show the mean number of correct responses for the determinate problems. In half of the problems the correct response was "yes" (hit), in the other half it was "no" (correct rejection). The right two bars show how often the participants accepted a conclusion that hold in the model built by rule 1^{ins} or rule 1^{alt}, respectively. Error bars indicate standard errors.

ANOVA of the percentage of correct responses yielded a significant main effect of conclusion acceptance [$F(2,32) = 54.79$, $p < .01$]. Percentage of correct responses of determinate/valid and determinate/invalid items did not differ ($p > .75$). The high percentage of correct responses for the determinate items ($M = 92.19$; $SD = 11.14$) indicate that the participants understood the task and were able to perform well. Because the determinate items were easiest constructed from the left to the right for both cost functions we assume that they were indeed constructed from left to right. We also assume that the indeterminate items were constructed from the left to the right as well, since the decision has to be made directly after reading the second premise before knowing whether the item is determinate or indeterminate. We find a higher percentage ($t(34) = 5.49$; $p < .01$) of yes-answers for the items where the conclusion was true if the model was built by rule 1^{ins} than for items where the conclusion was true if the model was built by rule 1^{alt} (see Figure 1). This indicates that indeed the rules derived from our cognitive cost functions are more often applied than the alternative rules derived from the classical computer science cost function.

ANOVA of the reaction times of correct responses also yielded a significant main effect of conclusion acceptance [$F(2,18) = 4.25$, $p < .05$]. Reaction times for determinate/valid items ($M = 3618$ ms, $SD = 1427$) were significantly lower compared to determinate/invalid items ($M = 4887$ ms, $SD = 2691$; $t(34) = -4.67$; $p > .01$). Reaction times for indeterminate/rule1^{ins} items ($M = 4156$ ms, $SD = 3066$) were significantly lower compared to indeterminate/rule1^{alt} items ($M = 5057$ ms, $SD = 3457$; $t(20) = -2.29$; $p > .05$). This implies that conclusions of the determinate/valid items were easier to confirm than the ones of the determinate/invalid items and the conclusions of the indeterminate/rule1^{ins} items were easier to accept than the ones of the indeterminate/rule1^{alt} items. These easier items were those where the confirmation could easily be made by following the implicit direction of the queue provided that the queue was indeed constructed from left to right.

Other evidence

Further evidence for our model comes from the experiments of Jahn et al. (2007). Their participants inserted an object to an existing array, as opposed to adding it to one end of the array, more often for objects that would have been added to the left end of an array than for entities that would have been added to the right end of an array (Jahn et al., 2007, Experiment 2, Table 4). The authors come to the conclusion that: "Given that the participants constructed arrays from left to right, they evidently found it easier to add a new entity to the right-hand end of an array than to the left-hand end of an array [...]." (Jahn et al., 2007, p. 2081)

For a queue that is constructed from left to right our model predicts this behaviour, since rule 1^{ins} is applied to the objects inserted on the right of a reference object and rule 2^{ins} is applied to objects inserted on the left of a reference object. So the results of Jahn et al. (2007) confirm the predictions of our model.

Discussion

We introduced an approach how spatial reasoning can be modelled as verbal reasoning. The main idea is that the deduction process does not necessarily require deduction-specific mechanisms to operate on internal representations. Instead we assume that a simple order of objects (represented by words) and some genuine verbal cognitive mechanisms might guide the reasoning process. Following Polk and Newell (1995) we assumed that the cognitive processes in deductive reasoning can be based upon the same processes as language comprehension and generation.

From our point of view our approach is a helpful addition to the long lasting controversy between models and rules in reasoning (e.g. Johnson-Laird, Byrne, & Schaeken, 1994; Rips, 1994; Hagert, 1984). In fact, models are often identified with visuo-spatial processing and rules with linguistic or sentential mechanisms (e.g. Goel, Buchel, Frith, and Dolan (2000)). Our study, however, shows that this distinction does not reflect the actual differences between the two approaches. In fact, our approach is a model-based approach, because at no point during the inference process rules of inferences are used, instead the new information must be derived from the queue - the model. And our results suggests that such models can be the basis of verbal reasoning, so no visuo-spatial process are involved in the inference.

Our work has also shown that the approach and the related cost measure leads to good predictions about what kind of model will be created. It predicts behaviour better than the classical computer science approach to cost calculation. But there remain some open questions about the cost measure. One problem of our approach results from the fact that it is easier to move through the queue than alter the existing links, no matter how far we have to move. Another possibility is that if the queue becomes larger there might exist a critical distance when it requires more mental effort to move this distance through the queue than altering a link. This would imply that if the queue reaches a certain number of objects, new objects would not necessarily be attached to the end of the queue any more.

Another question is whether the starting point of a queue is really a link like all the other links in the queue. However, since this link is different concerning its cognitive nature it might be weaker or stronger than the links between object in the queue.

A third limitation of our project is that we only used problems with two premises, although we believe that the postulated rules also apply if there are more than two premises and three objects, as long as the premises all contain relations describing the same dimension. And it is possible to mix relations of the same dimension such as left and right, as done in many experiments (Jahn et al., 2007; Ragni, Fangmeier, Webber, & Knauff, 2006).

Finally, we postulate that the implicit direction of a queue can be chosen freely. But what is this choice based on? In a behavioural experiment, in which many spatial reasoning problems need to be solved, subjects are likely to choose the direction that produces the lowest cost for the items seen so far. Also, once a choice has been made on which direction to use, subjects are likely to stick with it. This also keeps the mental costs low because the tactic used is not constantly being analysed. Also, when using material with a left-right dimension as in the examples there seems to be a general preference for constructing a left to right queue (Jahn et al., 2007; Rauh et al., 2005; Ragni et al., 2006). This could be a cultural preference such as reading. Also other orders we see in daily life are arranged left to right.

Overall, we were able to present some evidence for our assumption that the process of constructing a “verbal mental model” from premises influences deductive spatial reasoning. The chosen interpretation for the implicit direction of the queue has consequences on what kind of conclusions can be easily made. And, most importantly, for indeterminate problems, we can predict which model is preferred over the other and and which model is more difficult to consider as a possible interpretation of the premises. While our model can not necessarily be generalized to other domains of reasoning we feel that it can describe some aspects of human reasoning with spatial relations and that it demonstrates that spatial reasoning can also be conceived as verbal reasoning.

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