Placeholder structure and numerical computation

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Keywords: Placeholders, number, counting, gesture.

Introduction

This symposium explores the role of placeholder structures—systems of words, non-linguistic symbols, or procedures—in the construction and manipulation of numerical concepts. The structure supplied by a placeholder system – like the count list in English – critically constrains the potential for creating and manipulating conceptual content.

A great deal of empirical work has explored the role of placeholders in numerical cognition, in large part by investigating how children learn number word meanings when exposed to Arabic numerals and corresponding count routines (for review see Carey, 2009). One contribution of this work has been to suggest that learning a count list creates an important structure for constructing new numerical concepts. For example, learning to count is a critical precursor to acquiring large exact numerical concepts like "77", and how such concepts are related (e.g., that 78 is greater than 77, by exactly one). According to some, this learning is guided by a semantic induction, whereby children realize that each successive numeral in the count list denotes a quantity of 1 more than the number that came before it.

Beyond these studies, relatively little work has tested the role of placeholder structures outside the Arabic numeral system. As a result, little is known about the role that this particular structure plays in numerical development, and whether the use of alternative systems might result in different conceptual outcomes. Perhaps the best evidence that placeholder structures are critical to constructing numerical concepts comes from fieldwork in the Amazon, where studies of the Piraha and Munduruku have made clear that learning to count is important to acquiring at least some numerical concepts (Gordon, 2004; Frank et al, 200; Pica et al.). However, between the Piraha and English-speaking adults who use Arabic numerals lies a vast array of potential intermediate systems. At least some of these systems have been tested in nature by humans, and are used today. This symposium explores the role of placeholders systems to numerical development, the effects of structural variations, and how different modalities like gesture and vision are used to create alternatives to the Arabic numeral system, with different consequences for cognition.

Specifically, the symposium will include four distinct talks, each touching on different systems of numerical representations. Each talk will discuss how placeholder structures guide and constrain learning, whether by facilitating the association of symbols with quantities, guiding inductive inferences, or facilitating operations that are unique to a particular structure.

Number word meanings and the count routine

What role does a placeholder system like counting play as children learn number word meanings? In this talk, Barner will explore the idea that counting provides one of several verification procedures that children acquire when learning number words. Acquiring these procedures does not alone result in conceptual change, but instead lays the groundwork for learning about quantity and the logical relations between numbers. In particular, Barner will discuss how learning about the structural relationship between words in the count list may allow children to derive the concept of exactness, without a radical conceptual change, but instead drawing on well-attested pragmatic inferences. Barner also explores the so-called "Cardinal Principle induction" and whether it involves a conceptual change, or whether it is instead another example of procedural learning.

Number knowledge in a finite counting system

In this talk, Frank presents research examining a linguistic number representation used by a group of indigenous speakers of the language Momu (also known as Fas), spoken in the northern part of Papua New Guinea near the Indonesian border. The Momu count list has been reported to have a simple pair-based compositional structure that can be glossed as "one" (1), "two" (2), "two and another" (3), "two two" (4), "two two and another" (5), and "two two two" (6). The Momu count list is a fascinating case study of the relationship of placeholders to numerical competence.

Most Momu speakers had difficulty completing exact quantity matching tasks, failing to use linguistic number to track the quantity of objects presented by an experimenter. Even more surprising, Momu speakers did not agree on the structure of the Momu count list. Some speakers were able to count recursively to ten using the pair-based structure described above, while others claimed that the system was finite and bounded at "two and another" (3). The participants that did best on the matching tasks used the pair-based linguistic strategy, but not all participants that counted recursively applied the count in the matching tasks.

Momu is thus a case of dramatic linguistic and conceptual heterogeneity. Unlike English, where conceptual knowledge is deep and uniform across speakers, and unlike the Amazonian examples with essentially no exact number system, Momu speakers are on the cusp of knowledge: they know what they do not know, but do not have routines or strategies to complete even simple matching tasks.

The origin of numbers as summary symbols: Evidence from home sign and Nicaraguan Sign Language

Remembering a list of 9 items is harder than remembering a list of 6, but remembering the number "9" is no more difficult than remembering the number "6". As a result, numerals allow us to represent multiple individuals without adding costs to memory as a function of set size. This talk, by Spaepen, asks whether finger representations are summary symbols for entire sets (like "6") or for the individuals within that set (like 6 separate items).

Five signers of Nicaraguan Sign Language (NSL) and 4 unschooled hearing adults were tested using a modified digit span task, in which any one span only contained two numbers in an ABA pattern. There were 3 types of trials: patterns using 2 and 3 (e.g., 2, 3, 2), patterns using 4 and 5 (e.g., 4, 4, 5), and patterns using 8 and 9 (e.g., 9, 8, 8). Both groups performed equally well on all trial types, suggesting that both spoken words and conventionalized finger representations of number can act as summary symbols. Nicaraguan homesigners (deaf individuals who have no access to conventional linguistic input, spoken or signed, and who develop gestures systems to communicate with the hearing people around them) were tested on the same task and performed significantly worse on the 8 and 9 trials than on the other two trial types.

The NSL signers' performance reveals that finger representations can be summary symbols of the numbers they represent, and therefore can be placeholders for exact number concepts during development. However, when gestures for number are not learned in a rote list during language development, finger gestures represent individuals in the set, not whole sets. Because of this, homesigners' gestures may be used as placeholders, as they are not symbols that mean "seven," but rather symbols that mean "one one one one one one one."

The role of gesture in supporting visual representations of number

Mental abacus calculation is one of the most efficient methods for solving arithmetic problems mentally. Rather than physically moving the beads on an abacus, mental abacus experts memorize the operations necessary to move the beads and keep track of the current state of an imaginary abacus using visuospatial working memory (Stigler, 1984; Hatano, 1977). The abacus serves to represent the aspects of number necessary to compute basic arithmetic, while allowing these computations to be carried out by rote, rather than depending on detailed conceptual representations of each step. While performing mental abacus, nearly all abacus users move their hands as though they were manipulating an actual abacus. Past research has found that performance suffers when abacus users are not permitted to use their hands (Frank & Barner, under review; Hatano, 1977). Thus, gesture appears to play a critical role in creating and sustaining mental abacus structures. In this talk, Brooks will present work that explores the precise relationship between gesture and the structure of mental abacus computations.

A series of studies of mental abacus students in Gujarat Province, India, investigated the relationship between gesture and mental abacus. In addition to showing a powerful overall motor interference effect, this work shows an effect of an individual's default gesture size on the degree to which their performance suffered on the interference task. Children who spontaneously produced larger gestures when solving mental abacus problems showed a greater decline in performance when they were not permitted to gesture. In a second study, manipulating the size of a child's gestures led to changes in mental abacus performance: in general, instructing children to imagine a small abacus, and to gesture accordingly, led to better accuracy and reaction time compared to when children were instructed to imagine a large abacus. Further, the data suggest that preference for a smaller abacus size may be mediated by the size of gestures children produce spontaneously.

While research presented in this symposium and elsewhere (Carlson, Avraamides, Cary, & Strasberg, 2007) has demonstrated the important role of gestures as placeholders during counting, this work illustrates the dynamic role gesture can play in supporting and shaping complex computational systems in the visual domain.

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