

# When Choice Effects Compete: An Account by Extended EBA Model

Kenpei SHIINA (shiina@waseda.jp)

Department of Educational Psychology, Waseda University, Tokyo, Japan

## Abstract

A modified version of EBA is proposed to account for choice set effects (similarity, attraction, and compromise) and their interactions. The new model has ingredients of search-for-dominance-structure theory and counter race model, highlighting conflict resolution and deliberation in decision making. Simulation results show that the model can reproduce the choice set effects and predict the interactions between them.

**Keywords:** EBA; Choice Set Effects; Dominance Structuring ; Race Model.

## The EBA Model Revisited

The elimination by aspects model (EBA, Tversky, 1972a, b) is a classical and well-known model, which remains attractive because it was one of early models that provided a clear processing assumption for choices with mathematical rigor. EBA model asserts that an aspect is probabilistically selected and options that do not have this aspect are discarded. (In this paper, attribute, aspect, and feature are used interchangeably). Another aspect is then selected and options that do not have the second aspect are discarded. Proceeding in this way, the EBA process terminates when only one option remains. Although EBA is a simple model that does capture an important facet of decision making, there are many empirical effects and theoretical notions that the model cannot explain. This paper describes an updated version of EBA model that can account for choice set effects (e.g., similarity, attraction, and compromise effects) and their interactions.

The paper consists of four parts: The first section briefly reviews properties of EBA in view of theories and findings after its proposal in 1972. In the second section, a modified EBA called REGAL model is proposed (Shiina, 1994, for an earlier version): the model incorporates the ideas of dominance structuring and race model. In the third section, a simulation study is presented that shows how the new model explains choice set effects and their interactions by producing probability topographies. Finally, implications for further research on choice and decision making are addressed.

## Original EBA and its Properties

We use the following symbols and notations (Figure 1a).

$T = \{X, Y, Z, \dots\}$ : The finite total set of choice alternatives.

$\Omega = \{\alpha, \beta, \gamma, \rho, \dots\}$ : The finite total set of features (aspects).

$x', y', z', \dots \subseteq \Omega$ : Subsets of  $\Omega$  representing X, Y, Z, ...

For example, if X has features,  $\alpha, \rho, \omega$  and  $\theta$ , then

$x' = \{\alpha, \rho, \omega, \theta\}$ . It is assumed that  $\Omega = \bigcup_{x' \in T} x'$ .

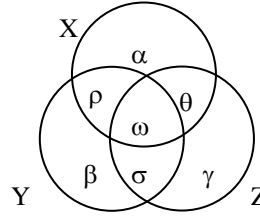


Figure 1a

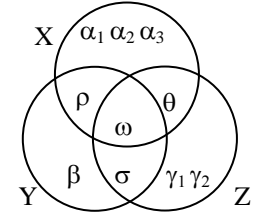


Figure 1b

## Common Aspects

Aspects that are common to all the alternatives ( $\omega$  in Figure 1a) should be ignored in EBA whereas there is evidence that common aspects play an important role both in decisions and choice satisfaction (Chernev, 1997). It seems very unnatural, moreover, that we should intentionally ignore the important features, common or not, that come to mind. If we consider a version of EBA that permits common aspect selection (Common aspect EBA or CEBA), it can be proved that CEBA does not change choice probabilities and can produce decision time predictions. Suppose that  $T = \{X, Y, Z\}$ ,  $x' = \{\alpha, \rho, \theta, \omega\}$ ,  $y' = \{\beta, \rho, \sigma, \omega\}$ , and  $z' = \{\gamma, \sigma, \theta, \omega\}$  (Figure 1a) with the understanding that  $\alpha = u(\alpha)$  etc., where  $u()$  is a utility (value) function. In binary case:  $T = \{X, Y\}$ , we have

$$P(X \leftarrow \{X, Y\})^{EBA} = (\alpha + \theta) / (\alpha + \theta + \beta + \sigma)$$

$$P(X \leftarrow \{X, Y\})^{CEBA} = (\alpha + \theta) / L + (\rho + \omega) / L \times P(X \leftarrow \{X, Y\})^{CEBA},$$

$$\text{where } L = \alpha + \theta + \beta + \sigma + \rho + \omega$$

Solving the second expression for  $P(x \leftarrow \{x, y\})^{CEBA}$ , we have

$$P(X \leftarrow \{X, Y\})^{CEBA} = \frac{(\alpha + \theta) / L}{1 - (\rho + \omega) / L} = \frac{(\alpha + \theta) / L}{\{L - (\rho + \omega)\} / L}$$

$$= (\alpha + \theta) / (\alpha + \theta + \beta + \sigma) = P(X \leftarrow \{X, Y\})^{EBA}$$

This equivalence also holds true for trinary or more numerous choices and thus Tversky's exclusion of common features is well grounded. For example, in trinary case:

$$T = \{X, Y, Z\},$$

$$P(X \leftarrow \{X, Y, Z\})^{CEBA} = P(X \leftarrow T)^{CEBA} =$$

$$1 / K \times \{\alpha + \rho P(X \leftarrow \{X, Y\}) + \theta P(X \leftarrow \{X, Z\}) + \omega P(X \leftarrow T)^{CEBA}\}$$

$$= \frac{\alpha}{K} + \frac{\rho}{K} \left\{ \frac{\alpha + \theta}{\alpha + \theta + \beta + \sigma} \right\} + \frac{\theta}{K} \left\{ \frac{\alpha + \rho}{\alpha + \rho + \gamma + \sigma} \right\} + \frac{\omega}{K} P(X \leftarrow T)^{CEBA}$$

where  $K = \alpha + \beta + \gamma + \rho + \sigma + \theta + \omega$ . This expression is identical to EBA if common feature  $\omega$  is ignored. Setting

$$L = \alpha + \rho(\alpha + \theta) / (\alpha + \theta + \beta + \sigma) + \theta(\alpha + \rho) / (\alpha + \rho + \gamma + \sigma)$$

we can derive a simple recursive formula:

$$P(X \leftarrow T)^{CEBA} = L / K + \omega / K \cdot P(X \leftarrow T)^{CEBA}$$

$$= L/K + \omega/K \left[ L/K + \omega/K \cdot P(X \leftarrow T)^{CEBA} \right] = \dots$$

$$= \frac{L}{K} + \frac{L}{K} \frac{\omega}{K} + \frac{L}{K} \left( \frac{\omega}{K} \right)^2 + \frac{L}{K} \left( \frac{\omega}{K} \right)^3 + \dots + \left( \frac{\omega}{K} \right)^m P(X \leftarrow T)^{CEBA}$$

Therefore when  $m \rightarrow \infty$ , we can derive an expression that is identical to the trinary EBA, because

$$P(X \leftarrow T)^{CEBA} = \frac{L}{K} \frac{1}{1 - \omega/K} = \frac{L}{K - \omega}$$

$$= \frac{\alpha + \rho(\alpha + \theta) / (\alpha + \theta + \beta + \sigma) + \theta(\alpha + \rho) / (\alpha + \rho + \gamma + \sigma)}{\alpha + \beta + \gamma + \rho + \sigma + \theta} = P(X \leftarrow T)^{EBA}$$

The generalization to  $n$ -option situation is straightforward, which gives reason for the idea that common features are processed but are indifferent to the final choice, and how the CEAB interpretation produces decision time predictions, because this interpretation yields a geometric distribution which permits rough estimates of decision times.

### Conjunction of Features and Dominance

It appears that EBA uses attributes one at a time. A careful reading of the original paper reveals, however, that an aspect can be an aggregate or conjunction of aspects. The aspect  $\alpha$  in Figure 1a, for example, may itself be a set of sub-aspects  $\{\alpha_1, \alpha_2, \alpha_3\}$  as in Figure 1b. In Figure 1, the choice of  $\alpha$  automatically leads to the choice of X. This paper interprets this property as “X was chosen because X dominated other options on  $\alpha$ .” If  $\alpha = \{\alpha_1, \alpha_2, \alpha_3\}$ , the interpretation will be “X was chosen because X dominated the other options on  $\{\alpha_1, \alpha_2, \alpha_3\}$ ”.

Dominance or dominance structuring is another key concept in the present paper. According to Montgomery & Willen (1999, p.148), “the decision maker attempts to find a dominance structure, that is, a cognitive structure in which the to-be-chosen alternative dominates other alternatives on relevant attributes.” It seems natural and promising to interpret EBA within the framework of search for dominance structuring (SDS) theory (Montgomery, 1989; Montgomery & Willen, 1999).

Because SDS model asserts that dominance should be established on a *bundle* of relevant attributes, a to-be-chosen alternative should be dominant on the conjunction of relevant attributes. A comparison of EBA and SDS in this respect gives a novel perspective; EBA is a very limited SDS in the sense that EBA process is a type of dominance structuring based upon a single attribute or a bundle of unique attributes. The key ideas linking EBA and SDS models are that dominance structuring is performed on a *conjunction* of attributes and the conjunctive set is *sampled* from the total set of attributes and thus changes over time. The conjunction of attributes is called an *evaluation set* or an *aspect lineup* in the model to be presented. An evaluation set is a set of attributes, so it is totally different from the consideration set, which is a set of alternatives.

### Reconsideration, Deliberation, and Deferral

Decision deferral and deliberation are two sides of the same coin because, in both cases, the decision maker cannot resolve decisional conflict at hand and thus deliberation evolves over time. Whereas it is often said that there are no widely accepted definition of conflict (Tversky & Shafir,

1992), we adopt the simple definition of Coombs and Avrunin (1988, p.222) that conflict arises in “a situation in which a choice must be made in the absence of dominance”.

It is often argued that EBA is suitable for relatively easy everyday choices, but may lead to less than optimal decisions. Janis and Mann (1977, p.32) pointed out, for example, that the decision maker may run out of relevant aspects and/or alternatives before reaching a decision, or may end up taking an alternative that is inferior to those eliminated.

If a pure EBA processing is used, it is almost inevitable that we may arrive at a much inferior choice with some probability. In that case, it will be very hard to justify both the final outcome and the choice process leaving a strong feeling of regret. We may start over, reconsidering discarded attributes and alternatives, whereas the original EBA does not have the mechanism to allow for such reprocessing. It is obvious that we should deliberate when the decision problem at hand is very important (buying expensive items, choosing a spouse or a job, etc.). Deliberation is a time-developing process and can continue for months or even years, during which time we think about the choice repeatedly. In this regard, an EBA model reinforced by SDS seems very appealing, because deliberation and justification are the key concepts of the SDS theory. Further, the combination is consistent with the current consensus that preferences are actively constructed, not merely revealed (Bettman, Luce, & Payne, 1998).

### Conflict Resolution and Counter Model

Tversky himself later maintained that decision making is a type of conflict resolution and that justification for the decision is necessary (Simonson, 1989; Simonson & Tversky, 1992; Tversky & Shafir, 1992; Shafir, Simonson, & Tversky, 1993). Dominance structuring is one method of conflict resolution and, if successful, it makes the choice self-evident (Montgomery & Willen, 1999, p.148). Although there are easy decisions that can be made almost automatically, decision makers facing an important decision should repeat consideration. Therefore, a counter or race model (Smith & Van Zandt, 2000) may be hypothesized in which the confidence for each alternative is accumulated during reconsideration, as the decision problem is addressed from many perspectives over time. This reconsideration process, possibly with repeated generation of evaluation set by attribute sampling, would be necessary to make the decision satisfactory or satisficing.

### REGAL Model

The REGAL (REpeated-Generation of Aspect Lineup) model integrates concepts from EBA, SDS, and the race (counter) models and tries to extend EBA in the following ways. First, reconsideration of aspects and alternatives is allowed, as this is an undeniable aspect of real-world decision-making. Second, REGAL permits the generation of conjunctive aspects, because a decision maker often has a bundle of minimum but unstable requirements that can be represented by a fluctuating conjunction of aspects. Third,

REGAL does not ignore aspects that are common to the alternatives, because there is evidence that common aspects do affect choice by enhancing the satisfaction. Finally, the concepts of counter and criterion (threshold) are adopted from race model to allow for decision time predictions.

In short, our revised EBA incorporates a) *reconsideration and deliberation processes*, b) a *more flexible aspect selection* that permits conjunctions, c) the processing of *common features*, and d) decision time generation processes.

### The Flow of the REGAL Model

The processing flow of the model is as follows (see also Figure 2 and Appendix):

(1) Let  $T$  be the set of alternatives and  $\Omega$  be the set of attributes used to represent the alternatives. The set  $x' \subseteq \Omega$  is the feature representation of Alternative  $X$ .

(2) A decision maker probabilistically samples a set of attributes  $\Psi$  from  $\Omega$ , called an *evaluation set* or an *aspect lineup*. The evaluation set is repeatedly regenerated during each cycle of REGAL process as shown schematically by the loop from (6) back to (2) in Figure 2. The reconsideration processes are represented in the loop.

(3) The degree of satisfaction for each alternative is evaluated on the *current evaluation set* determined in (2). The degree of satisfaction  $0 \leq S_{\Psi}(x) \leq 1$  is defined as

$S_{\Psi}(x) = \text{Goodness of Option } X \times \text{Structural dominance of } X \text{ over the other alternatives on evaluation set } \Psi$ .

The “Goodness” part says when both the evaluation set and  $X$  have many features and thus are rich then the function tends to output a larger satisfaction value. The “Dominance Part” outputs how dominant  $X$  is over the other alternatives in the attribute structure induced by  $\Psi$ .

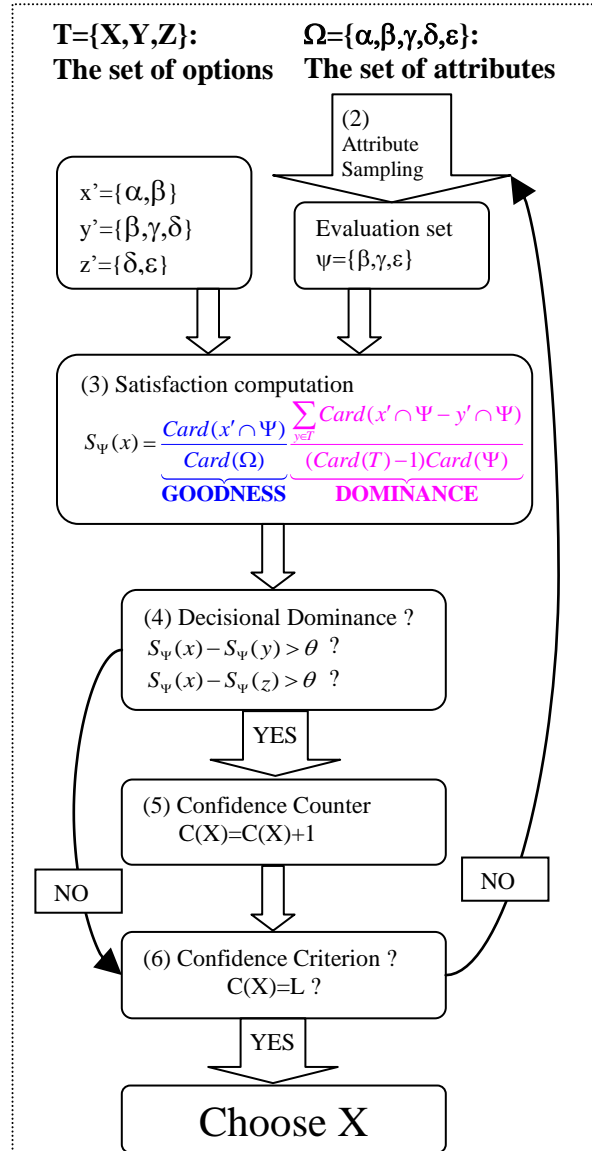
(4) A *decisional criterion*  $\theta$  is assumed. If  $S_{\Psi}(x) - S_{\Psi}(y) > \theta$  for all  $Y \in T - \{X\}$  then  $X$  is in the state of *decisional dominance*. If no option is dominant the process jumps to (6). Decisional dominance means that an option dominates the others in satisfaction at the moment.

(5) For each alternative, there is a *confidence counter*  $C(X)$ . If  $X$  becomes decisional dominant in (4), then a unit increment is added to  $C(X)$ . Race model is incorporated around (5) and (6).

(6) If  $C(X)$  reaches a *confidence criterion*  $L$ , then  $X$  is chosen else the process resumes from (2), where the regeneration of evaluation set  $\Psi$  is performed. The confidence criterion or threshold  $L$  represents decision quality: If the decision is momentous,  $L$  will be a high value, whereas if the decision is less important,  $L$  will be smaller. The depth of reconsideration processes are partially moderated by  $L$  and thus a merit of REGAL is that it encompasses the notion of decision quality.

When only one attribute is very important in decision, EBA, SDS, and REGAL tend to produce the same final choice.

**Counter Model** The loop between (6) and (2) constitutes a counter model (Smith & Van Zandt, 2000). The exact probability of choice is given in Appendix (Section 5).



**Figure 2: Outline of REGAL Model**

**Relation to Previous Models** Based on Manski (1977), several researchers have proposed conjunction models to make consideration sets (Andrews & Manrai, 1998; Gilbride & Allenby, 2004, 2006). *Subset Conjunction* proposed by Jedidi and Kohli (2005) is similar to the *evaluation set* in this paper. REGAL does not make consideration sets explicitly, but when  $y' \cap \Psi = \emptyset$  alternative  $Y$  is virtually excluded from the set of choice options and the remaining alternatives form a temporal consideration set. The major difference between REGAL and these previous models is that the consideration set is always temporary in REGAL.

### Simulation: Choice Set Effects

This section demonstrates how REGAL explains choice set effects (Roe, Busemeyer, & Townsend, 2001) and their interactions. Choice set effects have been studied on 2-dimensional continuous attribute spaces (Figure 3). Suppose

there are two options : A and B. Choice set effects (Similarity, Attraction, and Compromise effects) occur as a function of the location of new option C.

Both EBA and REGAL prefer discrete features and thus a special arrangement is needed to deal with continuous dimensions (See, Gensch & Ghose, 1992 for another approach). Basically, the continuous dimension is divided into segments and each segment is reinterpreted as a feature. For example, let  $A'=\{\alpha, \gamma, \delta\}$  and  $B'=\{\alpha, \beta, \gamma\}$  before the placement of C. If we place a new option  $C_1$ , the definition of segments and thus the features are changed accordingly (Figure 3) :  $A'=\{\alpha_1, \alpha_2, \gamma, \delta_1, \delta_2\}$ ,  $B'=\{\alpha_1, \alpha_2, \beta, \gamma\}$ , and  $C'=\{\alpha_1, \gamma, \delta_1\}$ , in this particular case.

**The Probability of Choosing a Feature** Let  $P(\tau), \tau \in \Omega$  be the probability that feature  $\tau$  is included in  $\Psi$ .  $P(\tau)$  is a function of feature importance or salience. In the present simulation, the length of feature would represent the salience, so that it is simply assumed :

$$P(\tau) = 1 - \exp(-\text{length}(\tau) \times K_1) \quad K_1: \text{constant}$$

which is sub-additive (Rottenstreich & Tversky, 1997) in the sense that  $P(\alpha) \leq P(\alpha_1) + P(\alpha_2)$  when  $\alpha = \{\alpha_1, \alpha_2\}$ .

**Value Function and Satisfaction Function** The value (utility) of feature  $\tau$  is again assumed to be a function of the length of feature :

$$v(\tau) = K_2 \times \log(\text{length}(\tau)) \quad K_2: \text{constant}$$

Using this function, the original binary version of satisfaction function (Equation (A.2) in Appendix) is converted into a continuous dimensional version:

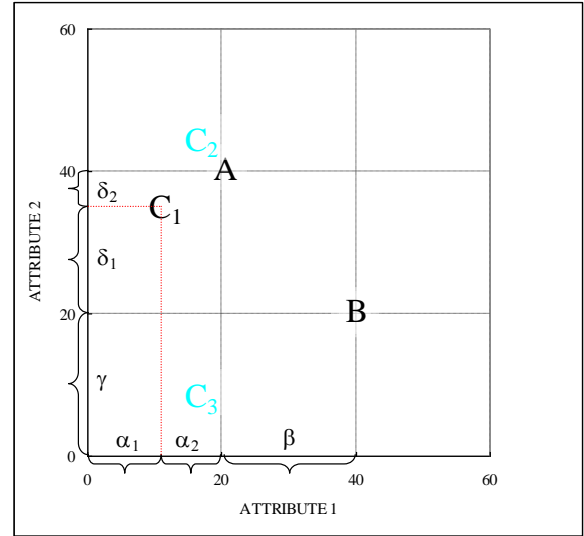
$$S_{\Psi}(x) \equiv \frac{\sum_{\tau \in x \cap \Psi} v(\tau)}{\underbrace{\text{Card}(\Omega)}_{\text{GOODNESS}}} \frac{\sum_{\tau \in x \cap \Psi - y \cap \Psi} v(\tau)}{\underbrace{(\text{Card}(T) - 1) \sum_{\tau \in \Psi} v(\tau)}_{\text{DOMINANCE}}}$$

**Interactions of Choice Set Effects** Suppose that new Option C is placed at  $C_1$ . Attraction effect predicts  $P(A)$  will be larger while Similarity effect predicts  $P(A)$  will be smaller. If C is placed at  $C_2$ , Similarity effect predicts  $P(A)$  smaller while Compromise effect predicts  $P(A)$  will be larger. If C is placed at  $C_3$ ,  $P(A)$  may become larger because C is dominated by both A and B but more strongly dominated by A than B, and there may be a slight similarity effect between C and B as well.

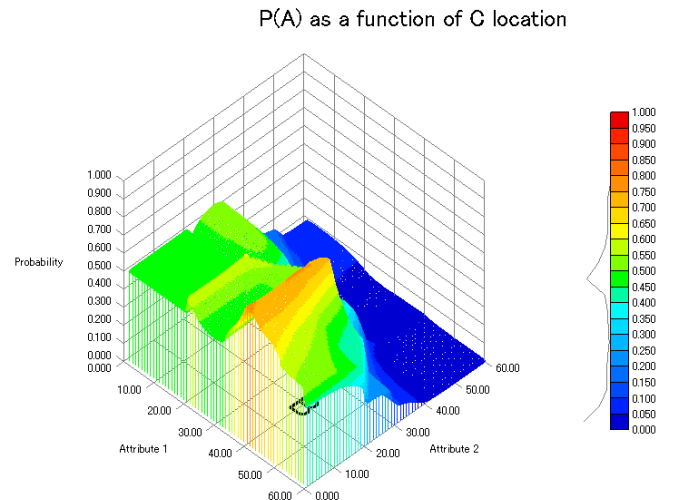
Apparently, in these cases choice set effects compete or collaborate and the choice probabilities should be determined as a non-linear function of *forces* that try to push up or down probabilities. In choice effect studies, *pure* conditions in which the effect of a single *force* becomes observable have been used. A next challenge should be to clarify the joint effects of choice set effects and the present study is the first such attempt.

## RESULTS

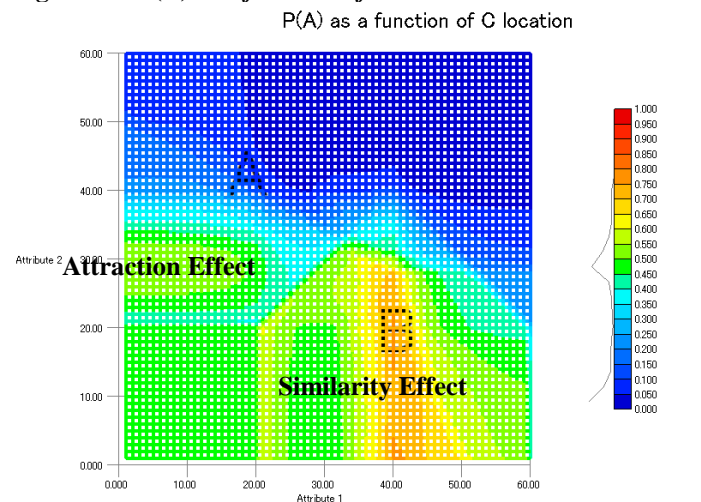
By moving Option C on the space (Figure 3), we can observe how the probabilities for Options A, B, and C change. REGAL can probe the joint effects by generating *probability topographies* (Figures 4, 5, and 6) calculated from Equation (A.3) in Appendix. Several detailed values for parameters are also shown in Appendix.



**Figure 3: Option representation on 2-dimensional attribute space.**



**Figure 4a: P(A) as a function of C location**



**Figure 4b: P(A) as a function of C location**

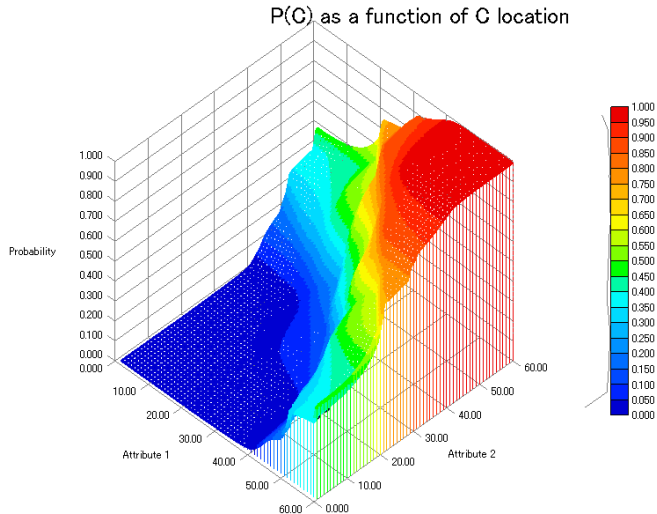


Figure 5a:  $P(C)$  as a function of  $C$  location

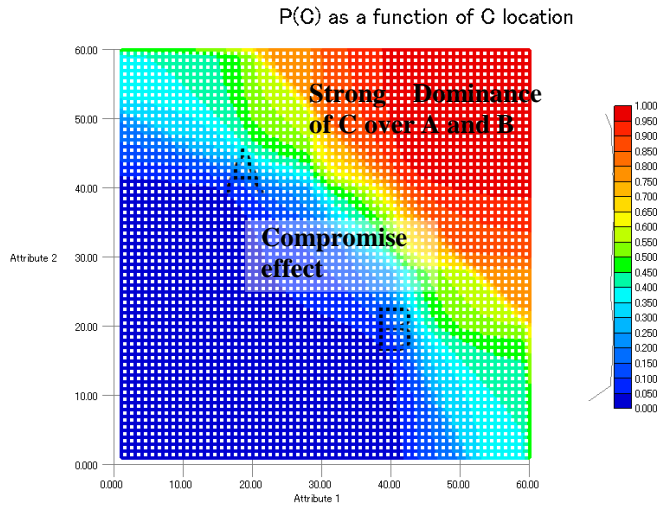


Figure 5b:  $P(C)$  as a function of  $C$  location

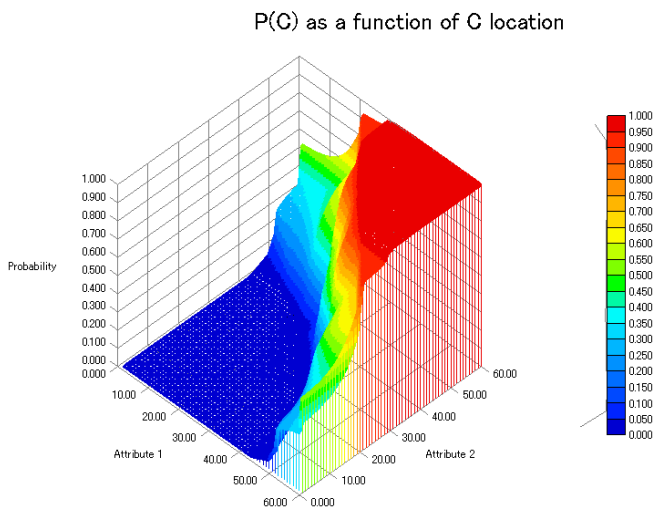


Figure 6:  $P(C)$  as a function of  $C$  location when  $L$  is large ( $L=20$ ).

Figures 4a (bird's eye view) and 4b (contour map) show the same topographic probability information. The probabilities are theoretical predictions in an ideal space

The reading of Figure 4a and 4b is a little confusing: they are showing  $P(A)$ , the probability that Option A will be chosen, as a function of  $C$  location. Therefore, the right lower high region, for example, should be read that “if you place  $C$  around here,  $P(A)$  will be high.” Figures 5a and 5b show  $P(C)$  as a function of  $C$  location. In this case, the figures give a natural interpretation. The graph for  $P(B)$  is omitted because it is an exact mirror image of Figure 4.

Major observations are as follows. 1) Strikingly high  $P(C)$  is obtained when  $C$  strongly dominates  $A$  and  $B$  (Figure 5b, upper right) and very low  $P(C)$  when  $C$  is dominated by both  $A$  and  $B$ . 2) Similarity effect is stronger than attraction effect (Figure 4b) and compromise effect is very weak (Figure 5b). Attraction effect was weak possibly because attraction and similarity effects compete in Figure 4b. 3) Increasing  $L$  made large probabilities larger and small probabilities smaller. As a result, the slope in Figure 6 became steeper. Psychologically, this would mean that deeper deliberation changes probabilistic choice into something akin to logical judgment, decreasing the chance of taking inferior options. Of course, these observations depend upon the present configuration of options, definitions of features, the shape of the value function, and the parameter values. More intensive search is needed to validate the model. Due to space limitation, RT predictions will be presented elsewhere.

## Discussion

It is well-known that MDFT model (Roe, et.al, 2001) and LCA model (Usher & McClelland, 2001) are able to mimic the three choice set effects. The present model is distinct both in architecture and in processing assumptions and can deal with, at least in theory, any number of options and attributes. Further, it can produce predictions for choice probabilities and decision times. This paper showed only qualitative validity of REGAL in an ideal theoretical space and empirical tests will be necessary in the future study.

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## Appendix: Technical Details of REGAL

This part should be read with reference to Figure 2.

### (1) Initial setting

$T$ : the set of alternatives,  $\Omega$ : the set of attributes

### (2) Evaluation set construction

$\Psi$ : Evaluation feature set

$P(\tau)$ : The probability that feature  $\tau$  is included in  $\Psi$ .  $P(\tau)$  is an increasing function of feature importance or salience.

The probability of generating evaluation set  $\Psi$  is

$$P(\Psi) \equiv P(\Psi \leftarrow 2^\Omega - \phi) = \frac{\prod_{\tau \in \Psi} P(\tau) \prod_{\tau \notin \Psi} (1 - P(\tau))}{1 - \prod_{\tau \in \Omega} (1 - P(\tau))} \quad (\text{A.1})$$

Important features tend to stay in  $\Psi$  and common features are not forcefully excluded.

(3) **Satisfaction function**  $S_\Psi(x)$  measures the degree of satisfaction and is a function of  $\Psi$  and  $X$ .

$$S_\Psi(x) \equiv \frac{\underbrace{\text{Card}(x' \cap \Psi)}_{\text{GOODNESS}}}{\underbrace{(\text{Card}(T) - 1) \text{Card}(\Psi)}_{\text{DOMINANCE}}} \quad (\text{A.2})$$

$\text{Card}(\ )$  is the cardinality of a set. *Goodness* part takes a value in  $[0,1]$ : the value of 1 is obtained when  $x' = \Psi = \Omega$ , that is, alternative  $X$  is perfectly dominating the other alternative in the sense it has all relevant attributes, and the value 0 is obtained when  $x' \cap \Psi = \phi$ , that is,  $X$  has no relevant attributes under the current evaluation set. *Dominance* part represents the structural dominance of  $X$  over the other alternatives. This part also takes a value in  $[0, 1]$ : the value of 1 is obtained when  $x' = \Psi$  and  $y' \cap \Psi = \phi$  for all  $Y$  except  $X$ , that is, alternative  $X$  is perfectly dominating the other alternative with respect to the current evaluation set, and the value of 0 is obtained when  $x' \cap \Psi - y' \cap \Psi = \phi$ , that is,  $X$  is totally dominated by the other alternatives under the current evaluation set.

(4) **Probability of decisional dominance** The probability that Alternative  $X$  becomes dominant,  $M_X$ , is defined as

$$M_X \equiv \sum_{\Psi \in 2^\Omega - \phi} P(X \text{ is decisional dominant} | \Psi) P(\Psi) \\ = \sum_{\Psi \in 2^\Omega - \phi} P(S_\Psi(X) - S_\Psi(Y) > \theta, \forall Y \in T - X | \Psi) P(\Psi)$$

where  $\theta \sim N(\mu, \sigma^2)$ : Decisional criterion that may fluctuate.

In the text, the variance is set to 0 and thus  $\theta$  is a constant.

(5) **Confidence counter  $C(\ )$** . The loop between (6) and (2) in Figure 2 can be captured by a race model and an alternative that first reaches  $L$  is chosen, where  $L$  is a confidence criterion. From the standard result of Poisson counter model (Smith and Van Zandt, 2000), the final choice probability is given in closed-form by:

$$P(X) = \sum_{j_1=0}^{L-1} \sum_{j_2=0}^{L-1} \cdots \sum_{j_{X=L-1}}^{L-1} \cdots \sum_{j_{n-1}=0}^{L-1} \frac{1}{\prod_{k=1}^n j_k!} \prod_{k=1}^n \left( \frac{\lambda_k}{\sum_{i=1}^n \lambda_i} \right)^{j_k} \left( \frac{\lambda_X}{\sum_{i=1}^n \lambda_i} \right) \quad (\text{A.3})$$

where  $n$  is the number of attributes and  $\lambda_k$ 's are Poisson strength parameters. Without loss of generality, we can set  $\lambda_k = M_k$ . The free parameters are  $L, \mu, \sigma^2$  and  $P(\tau), \tau \in \Omega$ . By adjusting them, we can examine whether REGAL can mimic the three choice-set effects. In the simulation,  $L=5, \mu=0$ , and  $\sigma^2=0$  were used.