

Category Change in the Absence of Falsifying Feedback

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Abstract

Many conceptual change theories posit that change occurs when the learner becomes dissatisfied with the current conception (Ohlsson, 2011; Strike & Posner, 1992). A necessary component of dissatisfaction is falsifying feedback. The present experiments investigate whether participants exposed to a novel method for eliminating the ability to directly falsify a misconception will still be able to recategorize compared to participants that can directly falsify. The results suggest that direct falsification of a misconception is not necessary for recategorization, and that direct falsification may slow the learning process. Implications are discussed.

Keywords: Learning, recategorization, feedback, non-monotonic change.

Introduction

Both common sense and past research have assumed that conceptual change in particular and non-monotonic cognitive change in general is driven by a person's dissatisfaction with his or her current conception (Ohlsson, 2011). Dissatisfaction is in turn caused by falsifying information and experiences that are inconsistent with the current conception. Without falsification a person would presumably lack motivation to change (Chi, 2005; Chi, 2008; Chi & Brem, 2009; Gopnik & Wellman, 2012; Slotta & Chi, 2006; Strike & Posner, 1982, 1992). But once dissatisfaction has set in, the learner is ready to search for an alternative conception (Elio & Pelletier, 1997; Strike & Posner, 1982, 1992; Chi & Ohlsson, 2005; Ozdemir & Clark, 2007). The theme of falsification first became dominant in the history of science via the works of Karl Popper and Thomas Kuhn, but it has since spread to all aspects of knowledge change.

For example, Strike and Posner's (1982) claimed that students in a science classroom must be dissatisfied with their current conception before they are ready to learn a new conception. Moreover, dissatisfaction must surpass the threshold at which accommodation supersedes assimilation. The threshold is surpassed by the accretion of falsifying pieces of information that accumulate until the discrepancy cannot be ignored.

Similarly, the Theory-Theory posits that the knowledge revision process takes place when dissatisfaction with the current conception reaches an individual's threshold for

conceptual change in the course of cognitive development (Gopnik & Wellman, 2012).

As a final example, the Categorical Shift Theory describes conceptual change as a process that requires one to abandon or reject prior misconceptions via the recognition of differences between two or more general categories (Chi, 2005; Chi & Brem, 2009). Failure to filter information through an existing knowledge base leads to dissatisfaction with the current conception. Dissatisfaction leads to a search for an alternative knowledge structure capable of accommodating the new information.

In short, these and other theories of cognitive change assume that dissatisfaction is a necessary prerequisite for cognitive change in children, students, and both lay adults and scientists. However, both common sense and psychological research agree that although people respond to falsifying information by trying to reduce the cognitive dissonance it causes, they tend to process the falsifying information in such a way as to minimize its impact on current knowledge (Ohlsson, 2011). If so, why should we believe that falsifying information is a necessary component of conceptual change?

In contrast to the theories mentioned above, the Resubsumption Theory claims that conceptual change can occur even in the absence of falsification of a person's current conception. This is possible when the learner possesses two alternative theories that apply to the same case or phenomenon. Change from one theory to the other occurs through competitive evaluation on the basis of *cognitive utility* rather than truth or falsity (Ohlsson, 2009). Competitive evaluation triggers a change by revealing that the alternative theory is more applicable in a given instance.

In the current study, we used the *re-categorization paradigm* (Cosejo, Oesterreich & Ohlsson, 2009) to create a situation in which the participants needed to change a newly learned definition a category into a different definition of the same category in the absence of information that falsified the latter. Specifically, the participants learned how to categorize a novel set of stimuli through the standard procedure used in countless categorization experiments (Ashby & Maddox, 2005): view a potential category member, judge whether it is a member, receive feedback on the judgment, and go to the next trial. Once the participants showed that they had mastered the category, the category was changed without warning. To succeed, the learner had

to re-learn the category, i.e., learn a new definition of it, and consequently, a different way of categorizing the relevant stimuli. The particular version of recategorization that we used in this study presented stimuli that mimicked a science-learning scenario. Images of fictitious alien bacteria were categorized with respect to their resistance to atmospheric oxygen; see details in the Method section.

The present study used the recategorization paradigm to investigate whether falsification is necessary for a learner to recategorize. All participants were given both supportive and falsifying feedback on their categorization judgments during the initial phase of the study. We refer to this as *initial learning*, and the category definition learned as the *initial category* or the ‘misconception’. After learning the initial category, the participants were exposed to one of two feedback conditions during the second phase of the experiment. We refer to the second phase as the *target learning*, and the new category definition acquired in this phase as the *target category*.

The participants in the complete feedback condition received both confirmatory and falsifying feedback (the *complete condition*). The participants in the second feedback condition were presented with stimulus items that had been altered in such way that the initial category, once acquired, could not be directly falsified (the *confirmation only condition*). This was accomplished by deleting crucial features from the stimuli; see Method section for details. However, they received the same information required to learn the target category as the participants in the complete condition. In short, the purpose was to compare recategorization in the presence and absence of falsifying feedback.

Predictions

There are three potential outcomes of this experiment. We could find that having complete feedback (i.e., both confirmation and falsification) yields the most efficient categorical change. Alternatively, we could find that the absence of falsification has no effect on recategorization, that is, learners need confirmation to learn, not falsification. Finally, we could find that falsification is not necessary, but harmful. That is, the presence of falsification might hinder recategorization, perhaps by creating cognitive conflicts that trigger defensive processing mechanisms (Ohlsson, 2011). The latter might use up cognitive resources that are needed for learning.

We have specific quantitative predictions regarding these outcomes. The predictions relate to different measures of performance. The first measure examines overall success, that is, do the groups learn the target when compared to chance. Specifically, it is hypothesized that the complete condition (i.e., those with both types of feedback) will perform better than chance because the combination of confirmatory feedback and falsifying feedback will allow the learner to adopt the target category. The confirmatory condition is hypothesized to perform better than chance because of the availability of confirmatory feedback.

The first measure (i.e., overall success) is examined between groups. That is, are there differences between groups in their ability to learn the target category? It is hypothesized that there will be no difference in target learning between the confirmatory and complete condition. This is expected because the use of confirmatory feedback will allow learners to adopt the target category (for both confirmatory and complete conditions). No differences between the confirmation and complete conditions will demonstrate that falsification is not necessary for recategorization to occur.

The second measure examines how quickly the groups can recategorize. There are three different scenarios that could occur for speed of categorization that will answer the question regarding what type of feedback appears to be the most effective for increasing speed of categorization. The first scenario would have complete learning faster than confirmatory. This would demonstrate that having both confirmation and falsification could result in faster learning compared to confirmation without falsification. That is, falsification is beneficial for increasing the speed of categorical change compared to not having the ability to directly falsify the misconception.

The second scenario would be that no difference exists between complete and confirmatory only conditions. This would suggest that the presence or absence of falsification has no effect on categorical change so long as confirmatory feedback is available.

The third scenario would show that speed of learning is faster for confirmatory compared to complete. This type of outcome would demonstrate that falsification might not be necessary for categorical change, but that it might hinder categorical change as evidenced by the complete condition underperforming compared to the confirmatory condition.

Method

Participants

One hundred twenty introductory psychology students participated in the study for course credit. Random assignment yielded 66 participants in the complete condition and 54 participants in the confirmatory condition.

Design

The study was a between-participants design with two conditions (Complete and Confirmatory).

Materials

The materials consisted of 128 fictional bacteria images including some that were incomplete, i.e., some features were deleted (see Figure 1). The bacteria have six different parts that have different binary attributes resulting in 64 complete variants: Nuclei (grey or black), Headbulbs (three or none), Ribosomes (bent or straight), Tail Cilia (present or absent), Cell Membrane (singular or double), and Cytoplasm (white or grey).

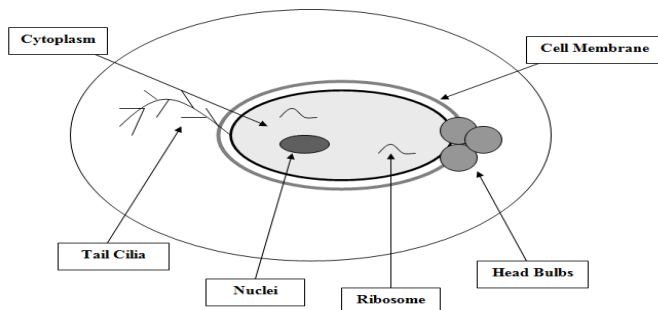


Figure 1. Example bacterium with parts labeled.

Additionally, some images were incomplete, that is, some images would not show the nuclei and some would not show the tail (see Figure 2). The images were presented on a computer screen via E-Prime software; see (www.pstnet.com/products/E-Prime/default/).

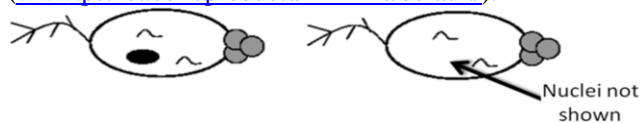


Figure 2. Bacteria with and without nuclei shown.

Procedure

Phase 1: Misconception Learning. Participants first learned to categorize whether an alien bacteria was oxygen resistant based on feedback that supported the misconception feature (i.e., *black nuclei*) over the course of five training blocks of 16 trials each. Each training block was balanced to include in randomized order six images that contained the misconception, six images that contained the target, two images that contained neither, and two images that contained both the misconception and the target. After five training blocks, unbeknownst to the participants the feature that determines oxygen resistance changed to *bent ribosomes* (i.e., the target).

Phase 2: Target Learning. Participants had five target training blocks of 16 randomized trials to learn that bent ribosomes determined oxygen resistance. The target training had two different experimental conditions.

Condition 1: Complete Stimuli. This condition consisted of stimuli that were similar to what participants had already used for classification. Each training block was balanced to include in randomized order six images that contained the misconception, six images that contained the target, two images that contained neither, and two images that contained both the misconception and the target. All parts of the bacteria were visible on the screen allowing participants to falsify their prior categorization in favor of a new categorization supported by the computers feedback. For example, in phase 1, the participant learned that black nuclei are responsible for oxygen resistance. In phase 2, the participant was then confronted with an image containing black nuclei with feedback stating that the bacteria was not oxygen resistant. This feedback should allow the learner to negate the prior conception. Moreover, when the learner is confronted with an image that does not have a black

nucleus, but is shown to be oxygen resistant the learner should logically conclude that another part of the bacteria is responsible for oxygen resistance.

Condition 2: Incomplete Stimuli. This condition contained no stimuli that could be used to directly falsify the misconception. Specifically, bacteria images containing the dark nuclei with straight ribosomes were not shown for any trial. However, there were stimuli that did not show the nuclei, resulting in an inability of the learner to directly falsify the initial category. Each training block was balanced to include in randomized order six images that did not display the misconception, six images that fit the target category, two images that fit neither category, and two images that fit both the misconception and the target categories. The purpose of the latter was to make the learning situation somewhat more challenging by introducing a small amount of noise into the information the participants received.

Procedure

Participants were seated in separate cubicles. Each participant was instructed to first participate in a training session, which consisted of a series of PowerPoint slides outlining how one can sort a variety of objects into different categories. The training session ended with participants categorizing stick figures based on their features. When participants finished with the initial training activity, they were instructed to participate in the more challenging bacteria paradigm.

Participants read the instructions for the task on the computer screen and asked questions if needed. Participants were given a script stating that alien bacteria was recently discovered on a distant planet and that scientists needed to determine whether there were oxygen resistant variants of the bacteria. Participants were then asked to rate how important each feature was in determining oxygen resistance on a 7-point Likert scale from 1 (Not at all) to 7 (Extremely). After rating the features, participants went through a prompt that described the importance of determining which bacteria were oxygen resistant. Each participant was tasked with determining whether the pictured bacterium was oxygen resistant. Participants indicated their response via the keyboard. The following responses were acceptable: y= yes, n=no, d= don't know. Participants would then receive immediate feedback from the computer either stating that the bacterium was or was not oxygen resistant. Participants were instructed to make as few errors as possible.

After completing all trials, participants were again asked how motivated they were to perform the task well and to rate the importance of different features in determining oxygen resistance on the same 7-point Likert scale as before. The participants keyed in an open-ended response about which features they thought determined oxygen resistance. They then went to the next screen which asked whether oxygen resistance was always determined the same way. Finally, participants answered demographic questions.

Results

Thirty-eight participants in the complete condition and 36 participants in the incomplete condition met the criterion for inclusion in analyses (i.e., correctly classifying 14 of 16 alien bacteria in any of the initial five training blocks). The inclusion criterion was chosen as a way to insure that we tested participants who were successful in learning the misconception. We wanted to examine whether falsification is necessary for adopting a new method of categorization for the participants who succeeded in learning the initial misconception feature, not whether falsification is necessary to learn the target category from scratch.

Learning Misconception

Our first analysis determined whether random assignment was effective at producing equivalent groups. In order to determine whether participants might differ in their ability to learn the misconception, we examined their performance on the first five blocks via a repeated measures analysis of variance (ANOVA) with Greenhouse-Geisser correction found a main effect for blocks, that is, regardless of condition, participants improved in performance from blocks 1 thru 5, $F(2.32, 166.84) = 66.76$, $p < .001$, $\eta^2_{\text{partial}} = .653$. There was no main effect of condition, $F < 1$, $\eta^2_{\text{partial}} = .008$ nor did groups differ at rate of learning, $F(2.32, 166.84) = 1.09$, *ns.*, $\eta^2_{\text{partial}} = .307$. These results suggest that the groups were equivalent in their ability to learn the misconception.

Learning the Target

Our next step was to assess whether the confirmatory condition learned the target in blocks 6 through 10. Performance of 14 out of 16 or greater on any of the blocks 6 through 10 was rated as successful learning of the target; we found that 29 of 36 (80.55%) participants correctly learned the target category. Whereas, if participants maintained the misconception for all trials they would have resulted in 0 of 36 participants demonstrating that they learned the target.

Using a chi-squared goodness-of-fit test we measured overall target acquisition (i.e., in general did learning occur yes or no) against a more stringent probability (i.e., chance at 50%). Specifically, the results revealed that the confirmatory condition's target acquisition was better than chance, $\chi^2(36) = 13.44$, $p < .001$. Similar results were found for the complete condition where 29 of 38 (76.31%) participants learned the target, $\chi^2(36) = 9.00$, $p < .01$.

Differences between Groups for Target Learning

We examined whether conditions differed in target acquisition via a chi-squared test-of-independence that showed that the groups did not differ in target acquisition, $\chi^2(36) = 2.90$, $p = .09$. This suggests that removing the ability to directly falsify the misconception does not hinder a learner's ability to adopt a new method of categorization.

Additionally, we examined potential differences in learning rate based on condition following the switch. That

is, we wanted see whether one group learned faster than the other. A repeated measure ANOVA with Greenhouse-Geisser correction was used to determine whether there would be a difference in performance following the switch from the misconception to the target for blocks 6 through 10. The analysis revealed a main effect for blocks showing that participants improved with training, $F(2.646, 190.477) = 75.01$, $p < .001$, $\eta^2_{\text{partial}} = .671$, and a main effect for condition showing that the confirmatory condition performed better than the direct condition, $F(1, 72) = 7.60$, $p < .01$, $\eta^2_{\text{partial}} = .096$. The interaction was significant, rate of learning was faster for the confirmatory condition than the complete condition, $F(2.646, 190.477) = 5.21$, $p < .01$, $\eta^2_{\text{partial}} = .146$. These results suggest that the confirmatory condition may result in faster learning of a new conception (see Table 1).

Table 1: The means and (standard deviations) for percentage correct for blocks 6-10.

	Block 6	Block 7	Block 8	Block 9	Block 10
Complete	40.63 (16.61)	64.64 (27.43)	72.86 (28.11)	81.58 (24.49)	86.06 (19.89)
Confirmatory	63.72 (16.89)	77.78 (19.33)	83.51 (20.27)	88.54 (16.06)	88.02 (20.51)

Overview of Response Type by Condition

In Figure 3, responses that are misconception consistent (MCR) or target consistent (TCR) separated by condition are shown by training block. MCRs were responding *no* on target bacteria and TCRs were responding *yes* on target bacteria. These response types are independent from each other because of the *don't know* response option. The figure shows how response tendencies changed when the feedback was altered to support the target within and between conditions.

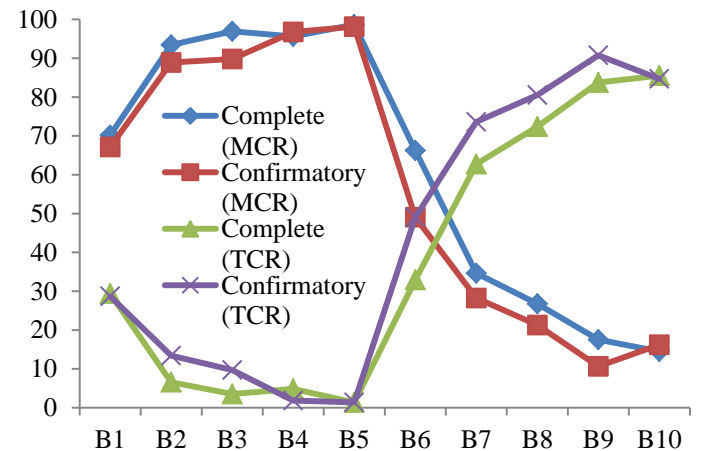


Figure 3. Percentage consistent with response type by condition.

Discussion of Experiment 1

The results of Experiment 1 suggest that the incomplete, confirmatory feedback only condition might initially speed

up learning of a new conception in comparison to the complete condition. We investigated whether participants would adopt a new conception after the switch and whether rate of learning would vary based on condition. We found that the participants in both conditions adopted the new conception. However, participants in the complete condition learned at a slower rate than those in the confirmatory condition. We propose that this difference was due to the need for participants in the complete condition to make sense of conflicting information. The sense making absorbed cognitive resources that otherwise would have been available for learning the target category, slowing down the re-categorization process.

Experiment 2

In Experiment 2, we attempted to replicate the findings of Experiment 1 with the addition of two learning aids. Learning aids were included in an effort to reduce the number of participants that are eliminated from analysis for failing to learn the misconception. The first learning aid was included in the prompt that participants read before engaging in the categorization process. The learning aid suggested that parts within the cell body may be influential in determining oxygen resistance (This statement is true as both misconception and target features are within the cell body). We assumed that the inclusion of this statement might focus search for what promotes oxygen resistance to the interior of the bacteria. The second learning aid was a handout that showed an image of the bacteria with parts labeled (see figure 1) as well as a list of the possible variants of each feature. This was meant to serve as a working memory aid.

Finally, the handout included a statement that “some of the images of the bacteria that you will see may be INCOMPLETE, that is, all bacteria have the 6 parts described, but some parts may not be visible.” This aspect of the handout was included in an effort to refute claims that participants may be viewing bacteria that do not show the dark nuclei as being bacteria *without* nuclei, which could result in a different interpretation of the stimuli by the participants.

Method

Participants

Sixty-one introductory psychology students participated in the study for course credit. Random assignment yielded 30 participants in the complete condition and 31 participants in the confirmatory condition.

Design & Procedure

The same procedure as in Experiment 1 was used in Experiment 2, with the addition of the two learning aids (i.e., the hint in the prompt and the handout).

Results

Twenty-one participants in the complete condition and 21 participants in the confirmatory condition met the criterion

for inclusion in analyses (i.e., correctly classifying 14 of 16 alien bacteria in any of the first five blocks).

Learning Misconception

Our first analysis sought to determine whether participants might differ in their ability to learn the misconception via a repeated-measures ANOVA where we examined percentage correct per block for the first five blocks. We found a main effect for blocks, that is, regardless of condition, participants improved in performance from blocks 1 thru 5, $F(4, 160) = 55.60$, $p < .001$, $\eta^2_{partial} = .582$. There was no main effect of condition, $F(1, 40) = 1.69$, $p = .201$, $\eta^2_{partial} = .041$ nor did groups differ at rate of learning, $F(4, 160) = 1.19$, $p = .319$, $\eta^2_{partial} = .029$. These results suggest that the groups were equivalent in ability to learn the misconception.

Learning the Target

Our next step was to assess whether the confirmatory condition learned the target in blocks 6 through 10. Performance of 14 out of 16 or greater on any of the blocks 6 through 10 was rated as successful learning of the target. We found that 15 of 21 (71.43%) participants correctly learned the target category. Using a chi-squared goodness-of-fit test the results revealed that the confirmatory condition’s target acquisition was better than chance, $\chi^2(21) = 3.86$, $p = .05$. Alternatively, results for the complete condition where 14 of 21 (66.67%) participants learned the target, their target acquisition was not better than chance, $\chi^2(21) = 2.33$, $p = .127$.

Differences between Groups for Target Learning

We examined whether conditions differed in learning the target via a chi-squared test-of-independence that showed that the groups did not differ in learning the target, $\chi^2(42) = .11$, $p = .739$. This replicates the finding from experiment 1 that the ability to directly falsify a misconception is not necessary for learning the new conception.

Given the relatively small sample size for Experiment 2 and the likelihood of differences occurring in earlier blocks we opted to conduct a series of t-tests on target learning blocks 6 through 10 instead of a repeated measure ANOVA, which might fail to differentiate the effect. The results of the t-tests revealed that participants in the confirmation condition performed better than the complete condition for block 6, $t(40) = 4.56$, $p < .001$ and marginally better on block 7 for a one-tailed t-test, $t(40) = 1.67$, $p = .051$. There were no differences between the groups for blocks 8, 9, and 10, $t < 1$ (see Table 2).

Table 2: The means and (standard deviations) for percentage correct for blocks 6-10.

	Block 6	Block 7	Block 8	Block 9	Block 10
Complete	44.94 (20.31)	68.75 (30.94)	78.87 (29.01)	80.06 (28.55)	83.93 (26.34)
Confirmatory	68.75 (12.66)	82.14 (19.89)	78.87 (16.11)	85.42 (19.8)	85.12 (23.67)

Discussion

In the present study, we examined whether participants randomly assigned to receive one of two types of stimuli differed in their ability to falsify an initially acquired category. The present findings provide modest support that there may be instances in which falsification is not only unnecessary for overriding a prior conception, but might actually be harmful. In both Experiments 1 and 2 we found that participants who could not directly falsify the misconception adopted the target conception in fewer trials compared to those participants who could falsify the misconception directly.

If replicated, our demonstrating that falsification is not necessary for categorical change could have multiple implications. For instance, theories of conceptual change that posit the necessity of dissatisfaction might themselves need revision. In addition, instruction in the classroom for scientific topics known to require knowledge revision has found that direct refutation is not necessarily effective at promoting change (Vosniadou, 1994; Vosniadou & Verschaffel, 2004), but perhaps novel development of another ontological structure could without refutation be developed and then integrated into the learning environment. Further investigation would be required in order to determine the most effective ways to improve conceptual change processing amongst students.

The present work should be viewed in consideration to its experimental controls, which might limit external validity. Specifically, the use of novel stimuli may not promote the same types of recategorical processes as stimuli that hold some greater individual meaning. Additionally, the population used in the study (university students) cannot be expected to adequately represent all types of learners. Furthermore, the learning processes observed in this experiment were of short duration. In many situations that require non-monotonic cognitive change, a direct verbal statement of the target concept is available but it was not part of our experimental procedure. Finally, we point out that conceptual change in real life usually involves a system of interrelated concepts rather than a single concept.

Future research might explore how different types of stimuli might influence recategorical change. Moreover, studies that mimic a classroom environment might also offer insights into what processes might bring about conceptual change. Additionally, studies that are able to use multiple daily training sessions and then attempt to recategorize might help in the understanding of temporal exposure and its influences on recategorization.

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