Are Fractions Natural Numbers, Too?

Percival G. Matthews (pmatthews@wisc.edu)
Department of Educational Psychology, 1025 W. Johnson Street
Madison, WI 53706 USA

Dana L. Chesney (dlchesney@gmail.com)
Department of Psychology, 1835 Neil Avenue
Columbus, OH 43210 USA

Nicole M. McNeil (nmcneil@nd.edu)
Department of Psychology, 118 Haggar Hall
Notre Dame, IN 46556 USA

Abstract

This study presents evidence in favor of a cognitive primitives hypothesis for processing fraction magnitudes. This account holds that humans have perceptual access to fractional magnitudes and that this may be used to support symbolic fraction knowledge. In speeded cross-format comparisons, participants picked the larger of two stimuli, which were either symbolic fractions or nonsymbolic ratios composed of pairs of dot arrays or pairs of circles. Participants demonstrated distance effects across formats, demonstrating that they could compare analog fractional magnitudes independently of the particular formats in which they were presented. These results pose a challenge to innate constraints accounts that argue that human cortical structures are ill-suited for processing fractions. These results may have important implications both for theorizing about the nature of human number sense and for optimizing instruction of fractional concepts.

Keywords: fractions; distance effects; number sense; numerical magnitude representations

Are Fractions Natural Numbers, Too?

What is so natural about natural numbers? At first glance, designating the counting numbers as ‘natural’ makes intuitive sense. Indeed, the natural numbers seem to play an important role in counting and numerical cognition more generally (see Noël, 2005). This "obvious intuition" may obscure the possibility that natural numbers are not alone in their ‘naturalness'. Here, we offer evidence that people may find fractional number values to be similarly intuitive.

A Cognitive Primitives Account of Ratio Processing

The proposition that fractional values may be intuitive might seem at odds with the fact that children often have considerable difficulties understanding symbolic fractions (e.g., Ni & Zhou, 2005). Many have argued that these well documented difficulties with fractions stem from innate constraints on the human cognitive architecture (e.g., Bonato, Fabbri, Umiltà, & Zorzi, 2007; Dehaene, 1997; Feigenson, Dehaene, & Spelke, 2004). Dehaene (1997) cogently summed up the gist of such innate constraints accounts when he wrote,

> Some mathematical objects now seem very intuitive only because their structure is well adapted to our brain architecture. On the other hand, a great many children find fractions very difficult to learn because their cortical machinery resists such a counterintuitive concept (p. 7).

Such accounts argue that the cognitive system for processing number, the approximate number system (ANS), is fundamentally designed to deal with discrete numerosities that map onto whole number values. Therefore, according to innate constraints theorists, fractions and rational number concepts are difficult because they lack an intuitive basis and must instead be built from systems originally developed to support whole number understanding.

Recently, Siegler and colleagues have sounded the call for researchers to reexamine the nature of fractional quantities, calling for a more integrated theory of numerical understanding that is inclusive of both natural numbers and fractions, rather than merely treating fractional values as an educational construct (Siegler, Fazio, Bailey, & Zhou, 2013). The present work seeks to answer that call. Contrary to the innate constraints hypothesis, here we put forward a cognitive primitives hypothesis for fraction magnitude processing. Emerging data from developmental psychology and neuroscience suggest that an intuitive (perhaps native) perceptually based cognitive system for grounding fraction knowledge may indeed exist. This cognitive system seems to represent and process amodal magnitudes of non-symbolic ratios (such as the relative length of two lines).

For instance, Vallentin and Nieder (2008) trained adult humans and monkeys on match-to-sample tasks using ratios between pairs of line segments. Monkeys and humans similarly performed far better than chance (85.5%), showing considerable sensitivity to fractional magnitudes. Moreover, using single-celled recordings, Vallentin and Nieder also found individual neurons that responded specifically to these visuospatial ratios constructed of line segments (e.g. a neuron that responded to ¼ in various instantiations). Other work has similarly demonstrated abilities of human infants, children, and adults to process nonsymbolic ratio magnitudes (e.g., Boyer & Levine, 2012; Jacob & Nieder, 2009; McCrink & Wynn, 2007; Sophian, 2000).

It remains a question whether symbolic fractions and nonsymbolic ratios actually access the same analog magnitude code. To date, there is no evidence demonstrating a link between this sensitivity to the magnitudes of nonsymbolic ratios and the acquired understanding for magnitudes of symbolic fractions; no research has shown that symbolic and nonsymbolic methods of presenting the same values converge to engage the same cognitive architecture for magnitude representation. We aimed to fill this gap by asking if cross-format comparisons of various fractional values (i.e. ratios composed of dots or circles vs. traditional fraction symbols) demonstrate distance effects. The distance effect – the phenomenon whereby error rates and reaction times vary inversely with increasing distance between the magnitudes of stimuli to be compared – is considered to be a hallmark of analog magnitude representation (Moyer & Landauer, 1967; Nieder, 2005; Schneider & Siegler, 2010; Sekuler & Mierkiewicz, 1977). For example, adult participants are faster and more accurate when choosing the larger digit in a forced choice of 2 vs. 9 compared to choosing the larger of 8 vs. 9. The existence of distance effects is generally taken to indicate an intuitive understanding of the magnitude of a given class of numbers.

Until recently, numerical distance effects were primarily investigated using symbolic whole numbers or their nonsymbolic analogs – numerosities. However, several recent studies have shown that human adults do indeed exhibit distance effects when comparing symbolic fractions (Jacob & Nieder, 2009; Kallai & Tzelgov, 2009, 2012; Meert, Grégoire, & Noël, 2010; Meert, Grégoire, Seron, & Noël, 2012; Schneider & Siegler, 2010; Siegler, Thompson, & Schneider, 2011). Critically, the cognitive primitives account predicts that different formats for fraction magnitudes should still converge on a single amodal representation of magnitude (cf. Balci & Gallistel, 2006). This account therefore predicts that there should be cross-format distance effects for comparing fractional magnitudes. With this experiment, we set out to test this key prediction. Currently, no studies have definitively shown distance effects using fractional stimuli across nonsymbolic and symbolic formats. Doing so would demonstrate that people can process and compare analog fractional magnitudes independently of their particular formats.

Table 1: Magnitudes and Components of Ratio Stimuli

Ratio Value	Dots	Circle (mm/mm)	Symbolic
.2	40/200	28/62	2/10
.3	60/200	34/62	3/10
.4	80/200	39/62	4/10
.5	100/200	44/62	5/10
.6	120/200	48/62	6/10
.7	140/200	52/62	7/10
.8	160/200	56/62	8/10

Note. Circle stimuli are listed in terms of diameter lengths of the numerators and denominators. Area ratios correspond to the ratios of the squares of these diameters.

The current experiment investigated the existence of distance effects using cross-format magnitude comparison tasks. In speeded tasks, participants picked the larger of two stimuli, which were either symbolic fractions or nonsymbolic ratios composed of pairs of dot arrays or pairs of circles. We predicted that we would find distance effects for cross-format comparisons similar to those typically found for within-format comparisons (Halberda & Feigenson, 2008; Moyer & Landauer, 1967; Odic, Libertus, Feigenson, & Halberda, 2013), indicating that participants have intuitive perceptual access to fractional magnitudes.

Method

Participants

55 undergraduate students at the University of Notre Dame participated for course credit (33 female; ages 18-22).

Materials and Design

All training and testing stimuli were presented on computers. Participants completed comparison tasks using paired ratios in different formats. There were three different stimulus formats: Arabic fractions, nonsymbolic dot ratios, and nonsymbolic circle ratios (see Figure 1). For each cross-format pairing, 98 comparison trials were created by taking all possible permutations of ratios corresponding to the magnitudes .2, .3, .4, .5, .6, .7, and .8 across stimulus pairs. Table 1 lists of all stimuli involved (described below).

Arabic Ratios Each symbolic stimulus was a fraction composed of an Arabic numerator and denominator separated by a fraction bar. Numerals for a given component were approximately 48.5 mm tall. We chose simple proper fractions because prior work has suggested that adult participants might have easily accessible analog representations of their values (Schneider & Siegler, 2010; but see Bonato et al., 2007 for an alternative take).

Dot Array Ratios Each stimulus was composed of a pair of dot arrays separated by a bar in the middle to form a non-symbolic ratio (Figure 1, Table 1). Non-symbolic numerosity arrays were composed of black dots on a white background. Displays were constructed controlling dot surface area so that all arrays had the same total surface area regardless of dot numerosity. However, individual dot size varied both within and between arrays, such that the size of a given dot did not precisely correlate with array numerosity. These controls mirror those that have been used in previous studies of numerosity perception (Hurewitz, Gelman, & Schnitzer, 2006; Xu, Spelke, & Goddard, 2005) To ensure that participants could not use computational procedures to estimate the ratios between these arrays, the smallest numerosity displayed in any given array was 40. This ensured that fast enumeration techniques, such as subitizing, could not be employed given the rapid stimulus presentation time (500ms, see Revkin, Piazza, Izard, Cohen, & Dehaene, 2008).

Circle Ratios Each stimulus was composed of a pair of circles separated by a horizontal bar to form a fraction (Figure 1, Table 1). Circles with 62.3mm diameters served as denominators for all stimuli. Numerator diameters varied to form the different area ratios.

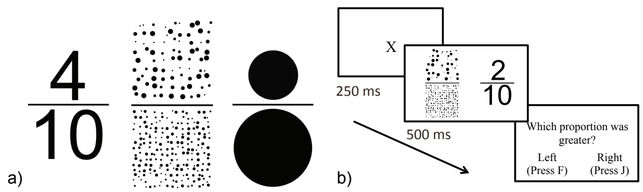

Figure 1: a) Sample symbolic, dot, and circle stimuli, each corresponding to the value of 4/10. b) A sample Arabic fraction vs. dot ratio comparison trial.

Procedure

Participants were presented with three blocks of comparison trials. Each block was composed wholly of trials of a particular cross-format pairing (i.e., Circle-Dot, Circle-Arabic, or Dot-Arabic). Block presentation order was randomized. Each block was organized identically so that participants first saw instructions, then received two practice trials, and then performed the experimental trials. Each participant completed all tasks in one hour-long session that also included other comparison experiments, including within-format versions of the comparisons described herein. Due to the current focus on cross-format distance effects, as well as space concerns, these tasks are not detailed in this report.

Instructions asked to participants decide which of two ratios was greater in magnitude. For circle stimuli, participants were specifically told to estimate “the ratios between circle areas or how much room each circle takes up on the screen.” For dot arrays, they were told to estimate the “ratio of the number of dots on top to the number of dots on bottom.” They were also told arrays would flash too briefly to count or to use calculations, so they should “just try to feel out the ratio instead of applying a formula.”

For each trial, a fixation cross appeared in the center of the screen for 250ms. The fixation cross was immediately followed by comparison stimuli, which were presented for 500ms. Once the stimuli disappeared, a prompt asked participants to indicate via keypress which ratio was greater – ‘f’ for the ratio on the left and ‘j’ if for the one on the right (see Figure 1). Participants saw 98 test trials in each cross-format block (one trial for each combination of stimulus pairs) for a total of 294 trials.

Results

We conducted two separate analyses for each of the three types of cross-format pairings (Arabic vs. Circle, Arabic vs. Dot, and Circle vs. Dot) to check for distance effects. The first analysis used summary scores of central tendency, paralleling analyses in previous research (e.g., Kallai & Tzelgov, 2009; Meert et al., 2012; Schneider & Siegler, 2010). Summary score regressions predicted mean error rates and median reaction times from inter-stimulus distance for each of the three combinations of stimulus pairings. We calculated distance for each comparison problem by subtracting the magnitude of the smaller stimulus from that of the larger stimulus. The logarithm of this distance (henceforth, log distance) was used as the independent variable in both error and reaction time regressions. Trials for which the distance was zero (e.g. the Arabic 3/10 vs. 60/200 dots) actually had no unique correct answers, so they were not included in the analysis. This analysis resulted in six data points per regression – one for each of the six possible distances (.1, .2, .3, .4, .5, and .6).

The second set of analyses was conducted using all raw data points instead of summary scores. These analyses were conducted to confirm that effects seen in summary score analyses were not artifacts of data loss inherent to collapsing aggregate data into summary indicators. In these regressions, we separately predicted error rate and RT from log distance. This analysis allowed additional controls to check for the symmetry of effects, so regressions included dummy coded control variables indicating a) whether the larger stimulus was presented on the left or on the right and b) which of stimulus types compared (e.g. dots or circles) was presented on the left. These controls, which are lost when we collapse across variables to create means and medians for summary score analysis, can provide insight into potential biases in participant behavior due to different aspects of task structure. Trials with RTs in excess of 4 SD above the mean were excluded from both RT and accuracy analyses, resulting in removal of < 0.6% of data points for each of the three types of comparisons. Only correct trials were included for RT analyses. All reported regression coefficients are standardized.

Summary Analyses

At the summary level, the log distance between fraction stimuli explained at least 98% of the variance in mean error rates and at least 83% of the variance in median RTs across all three types of comparisons (Table 2). Moreover standardized coefficients were large, indicating that a standard deviation decrease in logdistance led to nearly a standard deviation increase in errors and RTs for each of the comparison types. Consistent with our regression results, the graphs in Figure 2 show that both RTs and error rates decreased logarithmically as the distances between comparison stimuli increased. These distance effects closely paralleled those found by Schneider and Siegler (2010) for purely symbolic fraction comparisons.

Raw Data Analyses

Arabic Vs. Circle Stimuli The mean error rate for Arabic vs. Circle stimuli pooled across all trials was 17%, (*SD* = 38%), indicating that individuals were largely accurate when making comparisons across the two formats (see Table 2 for a summary of all raw data analyses). Log distance between stimuli was the only significant predictor of error rates (standardized β = -.29, p < .01). Mean error rates decreased from 32% to 2% as inter-stimulus distance increased from .1 to .6 (see Figure 2). There was no effect for which type of stimulus (circle or Arabic) was on the left (β = .01, p = .42) or for which side displayed larger fractional value (β = .01, p = .48). Similarly, log distance was the only significant predictor of RT, with RT also decreasing logarithmically as distance increased (standardized β = -.19, p < .01).

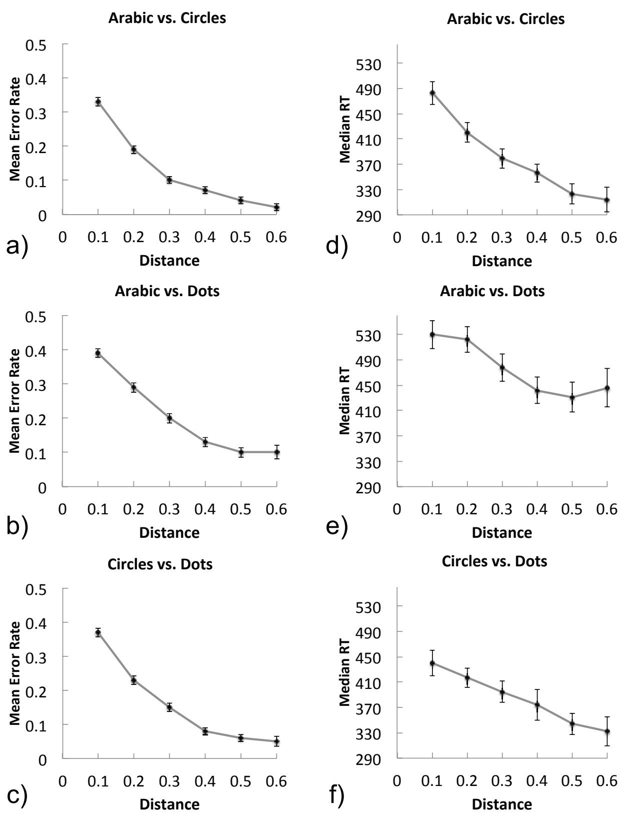

Figure 2: Graphs depict summary level data. Left panels (a, b, and c) show mean error rate as a function distance, and bottom panels (d, e, and f) show RT as a function of distance for the three types of cross-format comparisons. Error bars indicate standard error.

Arabic Vs. Dot Stimuli The mean error rate for Arabic vs. Dot stimuli pooled across all trials was 25% (SD = 44%) indicating that individuals were largely accurate on comparisons across the different formats. There was a significant effect of log distance, with errors decreasing as distance increased (standardized β = -.25, p < .01). Mean error rates decreased from 39% to 10% as inter-stimulus distance increased from .1 to .6 (Figure 2). There was also an unanticipated effect for the side on which the large stimulus appeared (β = -.07, p < .01). For Arabic vs. Dot ratio comparisons, participants were on average 6% more accurate when the larger stimulus was presented on the left compared to when the larger stimulus was on the right.

Table 2: Summary and Raw Data Analyses

	Summary Analyses		Raw Data Analyses	
	Mean Errors	Median RT	Error Rate	RT
Arabic vs. Circle				
Data Points	6	6	4595	3810
Adj R^2	.98	.99	.08	.04
β log distance	-.99**	-.99**	-.29**	-.19**
β large left	n/a	n/a	.01	.02
β Arabic left	n/a	n/a	.01	-.02
Arabic vs. Dot				
Data Points	6	6	4593	3439
Adj R^2	.98	.83	.07	.01
β log distance	-.99**	-.93**	-.25**	-.10**
β large left	n/a	n/a	-.07**	-.06**
β Arabic left	n/a	n/a	-.02	.01
Circle vs. Dot				
Data Points	6	6	4609	3651
Adj R^2	.98	.92	.09	.01
β log distance	-.99**	-.97**	-.29**	-.10**
β large left	n/a	n/a	-.02	.02
β Circles left	n/a	n/a	.00	.01

Note. All coefficients are standardized. Measures denoted n/a were unavailable for summary analyses do to collapsing across trials. ** p < .01

Despite this bias, standardized regression coefficients indicate that the effect size for log distance was more than three times as large the effects due to this bias. There was no effect for which type of stimulus (dot or Arabic) was displayed on the left (β = -.02, p = .23).

When RT data were analyzed, there was a significant effect of log distance, with RT decreasing as distance increased (β = -.10, p < .01). As with error analysis, there was an unanticipated effect for the side on which the large stimulus appeared (β = -.06, p < .01). Participants responded 51ms faster on average when the larger stimulus was on the left rather than the right. This left side bias, seen for both RT and error, may simply have been due to participants scanning the display from left to right. However, further study is needed to determine the cause of this effect. Nevertheless, the effect size for log distance was twice as large as the effect of side bias. There was no effect for which type of stimulus (dot or Arabic) was displayed on the left (β = .01, p =.86).

Circle Vs. Dot Stimuli The mean error rate for Circle vs. Dot stimuli pooled across all trials was 21%, indicating that participants were mostly accurate on comparisons across the two formats (SD = 41%.). As with the other stimulus pairings, there was a significant effect of log distance such that errors decreased as log distance increased (β = -.29, p < .01). Mean error rates decreased from 37% to 5% as inter-stimulus distance increased from .1 to .6 (see Figure 2). There were no effects for where the larger stimulus appeared (β = -.02, p = .22) or which type was on the left (β = .00, p = .84). There was also a significant effect of log distance on RT, with times decreasing as log distance increased (β = -.10, p < .01).

Discussion

Analyses using both summary measures and raw disaggregated data points consistently found distance effects both for error rates and reaction times for all three types of cross format comparisons. The full analysis of raw data points showed that distances persisted as significant predictors of performance, over and above noise due to individual differences. Thus, it seems that participants could access intuitive analog representations of fractional magnitudes, regardless of the format in which they were presented. Indeed, for both Arabic vs. Circle and Circle vs. Dot comparisons, distance was the only significant predictor of performance, and for Arabic vs. Dot comparisons, distance effects were two to three times larger than effects due to the (unanticipated and currently unexplained) 'large left' presentation bias.

This is the first study demonstrating such distance effects for fractional magnitudes across different notational formats. Participants completed these tasks both with stimuli composed of discrete non-symbolic numerosities (dot arrays) and with stimuli composed of continuous magnitudes (circle areas). Together, these performances provide evidence of flexible and accurate processing of nonsymbolic fractional magnitudes in ways similar to ANS processing of discrete numerosities.

Two potential concerns regarding this interpretation are a) the possibility that participants may have used conscious computation to convert the stimuli to a common format and b) that use of constant denominators within a particular stimulus format may have allowed participants to make comparisons solely using the numerators. However we were able to ameliorate these concerns to some extent by examining data from within-format comparison tasks run in the same experimental sessions as the cross-format tasks. First, cross-format comparisons on average took only 126ms longer than within-format comparisons using the same stimuli: differences ranged from 20ms (Dot-Circle: M = 478ms, SD= 378ms; Dot-Dot: M = 458ms, SD = 388ms), to 247ms longer (Arabic-Dot: M = 627ms, SD = 525ms; Arabic-Arabic: M = 375ms, SD = 335ms). These differences, though statistically significant, are substantively negligible as 126ms─or even 247ms─is insufficient time for computational conversions. Indeed, these RT differences are much *less* than the ~400ms conversion costs that have been reported for similar paradigms contrasting symbolic and non-symbolic whole number magnitudes (Lyons, Ansari, & Beilock, 2012). Second, RTs on within-format trials using matching denominators did not differ from those using non-matching denominators. Finally, preliminary results of a study we have run using varying denominators replicate the current findings.

These cross-format distance effects indicate that processing of ratio magnitudes bears the same signature that is typically found when other perceptual stimuli are used in comparison tasks. Our conclusions, accordingly, parallel those of Moyer & Landauer (1967) upon finding distance effects among Arabic numerals: "These results strongly suggest that the process used in judgments of differences in magnitude between [ratio values] is the same as, or analogous to, the process involved in judgments of inequality for physical continua" (p. 1520).

These results pose an important challenge to accounts that argue that basal human cognitive architectures are incompatible with fractions concepts (e.g., Dehaene, 1997; Feigenson et al., 2004). Considered in concert with other recent findings, our evidence suggests that humans may have an intuitive "sense" of ratio magnitudes that may be as compatible with our cortical machinery as is the "sense" of natural number. Just as the ANS allows us to perceive the magnitudes of discrete numerosities, this ratio sense provides humans with an intuitive feel for non-integer magnitudes. One important implication is that nonsymbolic ratios may function as cognitive primitives for supporting rational number concepts.

This work takes an important step toward advancing the cognitive primitive hypothesis, particularly in terms of describing some aspects of the human perception of ratio magnitudes. It also raises more questions for future research. One set of questions regard the nature of the link between ratio perception and the pedagogical process. For instance, Dehaene's (2007) charge about the incompatibility of neural structure with fractions was centrally concerned with the fact that learners often find it quite difficult to gain a correct understanding of symbolic fractions. If, as our results and others suggest, people come equipped to process fractional values, why do people – particularly novice learners – have such difficulties with symbolic fractions?

We suggest that these difficulties may result because the most common methods of teaching do not optimally engage the intuitive ratio processing system. For instance, the majority of current educational initiatives teach fractions as a sort of equipartitioning or equal sharing process that taps counting skills and understanding of whole-number magnitudes (e.g., Empson, 1999). It may be that these processes encourage counting and thereby discourage use of the ratio processing system. Indeed, past work has shown that young children perform worse on ratio matching tasks when partitioned, countable stimuli are used than when continuous stimuli are used (e.g., Boyer & Levine, 2012).

It remains to be shown how we can best leverage this perceptual appreciation for fractions to promote learning about fractional symbols. Siegler and colleagues (Siegler et. al, 2011; Siegler et al., 2013) have shown that an appreciation for fraction magnitudes is a crucial determinant of math achievement. It may be that the optimal way to foster such an appreciation is to use nonsymbolic ratio exemplars to inform learners about the magnitudes of symbolic fractions. By using perceptual exemplars that convey meaning about magnitude, we might eventually come to teach what a fraction like 1/3 represents in much the same way that we teach about what 4 represents. The current experiment did not address these questions of pedagogy because it focused on the performance of adults already conversant with conventional symbols for representing fractions, rather than that of novices. It does, however, open up a large space for future inquiry. For instance, are distance effects for visuospatial or symbolic fractions related to fraction knowledge test performance? How do these cross-format distance effects develop with age and experience? Answering this developmental question will be pivotal to evaluating whether or not the abilities examined in these experiments represent core competencies; it will determine how 'primitive' these cognitive primitives actually are. These and other questions await investigation.

More generally, research foregrounding our abilities to perceive ratios per se is in its infancy, and there remain many miles to go before we can draw conclusions about whether this sense of proportion is truly on par with – or perhaps even primary to – the sense for natural number that has been much more thoroughly investigated. What is certain, however, is that the study of ratio perception is full of possibilities. One such possibility is that fractions may be just as natural as natural numbers.

Acknowledgments

This research was partially supported by the Moreau Academic Diversity Postdoctoral Fellowship Program of the University of Notre Dame. We thank Michael Villano for his help in stimuli construction.

References

Bonato, M., Fabbri, S., Umiltà, C., & Zorzi, M. (2007). The mental representation of numerical fractions: Real or integer? *Journal of Experimental Psychology: Human Perception and Performance*, *33*, 1410–1419.

Boyer, T. W., & Levine, S. C. (2012). Child proportional scaling: Is 1/3 = 2/6 = 3/9 = 4/12? *Journal of Experimental Child Psychology*, *111*, 516–533.

Dehaene, S. (1997). *The number sense*. Oxford University Press, New York.

Empson, S. B. (1999). Equal Sharing and Shared Meaning: The Development of Fraction Concepts in a First-Grade Classroom. *Cognition and Instruction*, *17*, 283–342.

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences*, *8*, 307–314.

Halberda, J., & Feigenson, L. (2008). Developmental change in the acuity of the "number sense": The approximate number system in 3-, 4-, 5-, and 6-year-olds and adults. *Developmental Psychology*, *44*, 1457–1465.

Hurewitz, F., Gelman, R., & Schnitzer, B. (2006). Sometimes area counts more than number. *Proceedings of the National Academy of Sciences*, *103*, 19599–19604.

Jacob, S. N., & Nieder, A. (2009). Tuning to non-symbolic proportions in the human frontoparietal cortex. *European Journal of Neuroscience*, *30*, 1432–1442.

Kallai, A. Y., & Tzelgov, J. (2009). A generalized fraction: An entity smaller than one on the mental number line. *Journal of Experimental Psychology: Human Perception and Performance*, *35*, 1845–1864.

Kallai, A. Y., & Tzelgov, J. (2012). When meaningful components interrupt the processing of the whole: The case of fractions. *Acta Psychologica*, *139*, 358–369.

Lyons, I. M., Ansari, D., & Beilock, S. L. (2012). Symbolic estrangement: Evidence against a strong association between numerical symbols and the quantities they represent. *Journal of Experimental Psychology: General, 141*, 635-641.

McCrink, K., & Wynn, K. (2007). Ratio abstraction by 6-month-old infants. *Psychological science*, *18*, 740–745.

Meert, G., Grégoire, J., & Noël, M.-P. (2010). Comparing 5/7 and 2/9: Adults can do it by accessing the magnitude of the whole fractions. *Acta Psychologica*, *135*, 284–292.

Meert, G., Grégoire, J., Seron, X., & Noël, M.-P. (2012). The mental representation of the magnitude of symbolic and nonsymbolic ratios in adults. *The Quarterly Journal of Experimental Psychology*, *65*, 702–724.

Moyer, R.S., & Landauer, T.K. (1967). Time required for judgements of numerical inequality. *Nature, 215*, 1519–1520.

Ni, Y., & Zhou, Y.-D. (2005). Teaching and Learning Fraction and Rational Numbers: The Origins and Implications of Whole Number Bias. *Educational Psychologist*, *40*, 27–52.

Nieder, A. (2005). Counting on neurons: the neurobiology of numerical competence. *Nature Reviews Neuroscience*, *6*, 177–190.

Noël, M.-P. (2005). Finger gnosia: a predictor of numerical abilities in children? *Child Neuropsychology*, *11*, 413–430.

Odic, D., Libertus, M. E., Feigenson, L., & Halberda, J. (2013). Developmental change in the acuity of approximate number and area representations. *Developmental Psychology*, *49*, 1103–1112.

Revkin, S. K., Piazza, M., Izard, V., Cohen, L., & Dehaene, S. (2008). Does Subitizing Reflect Numerical Estimation? *Psychological Science*, *19*, 607–614.

Schneider, M., & Siegler, R. S. (2010). Representations of the magnitudes of fractions. *Journal of Experimental Psychology: Human Perception and Performance*, *36*, 1227–1238.

Sekuler, R., & Mierkiewicz, D. (1977). Children's Judgments of Numerical Inequality. *Child Development*, *48*, 630–633.

Siegler, R. S., Fazio, L. K., Bailey, D. H., & Zhou, X. (2013). Fractions: the new frontier for theories of numerical development. *Trends in Cognitive Sciences*, *17*, 13–19.

Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology*, *62*, 273–296.

Sophian, C. (2000). Perceptions of proportionality in young children: matching spatial ratios. *Cognition*, *75*, 145–170.

Vallentin, D., & Nieder, A. (2008). Behavioral and Prefrontal Representation of Spatial Proportions in the Monkey. *Current Biology*, *18*, 1420–1425.

Xu, F., Spelke, E. S., & Goddard, S. (2005). Number sense in human infants. *Developmental Science*, *8*, 88–101.