Perceived similarity mediates violations of independence in probabilistic judgments

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Abstract

We outline a simple way of representing sets of non-normative judgements that makes them look as similar as possible to normative ones. This representation allows us to view certain types of non-normative judgments, such as conjunction fallacies, as arising from a misestimation of the correlation between events, that might arise when decision-makers have no prior information about the frequency of co-occurrence. We suggest that decision-makers use the perceived similarity between events to make inferences about correlation, and we describe the results of an experiment showing that judged correlation and violations of independence in probabilistic judgments are strongly influenced by the perceived similarity between events.

Keywords: Conjunction fallacy, similarity, normative reasoning

Introduction

Being able to make unbiased and reasonably accurate likelihood judgments about simple events is a foundation for more complicated tasks such as inference or causal reasoning. Human reasoners are often very competent at providing such judgments, in the sense that their judgments align well with normative prescription. However there are classic results, such as those associated with the famous Tversky and Kahneman research program, which show that in some cases human reasoners may provide likelihood judgments for simple combinations of events that show systematic biases. In causal reasoning tasks for example, this can lead to violations of the predictions of models based on classical probability theory, with a resulting need to supplement classical models with extra unobserved relationships (Rehder, 2014), or even to reject classical probability theory entirely and attempt to construct models based on other theories of probability, such as quantum probability theory (Pothos & Busemeyer, 2013).

Several decades of experience have taught researchers where to expect violations of normative rules when making likelihood judgments, but there has been less success in determining why such violations occur. Tversky and Kahneman (1983) argued that conjunction fallacies occur because of a representativeness heuristic, while explanations for violations of normative rules such as the Markov Condition in causal reasoning often involve the presence of additional enabling or disabling causes (Rehder, 2014). Meanwhile models based on non-classical probability theory posit that violations of classical probability rules occur because of a somewhat mysterious property known as 'incompatiblity' (Busemeyer & Bruza, 2012). Perhaps one or more of these explanations is correct, perhaps none are, but regardless they all suffer from the same problem that it is difficult to predict in advance whether a given set of events will be 'representative' or 'incompatible' etc.

An additional problem faced when attempting to understand why some judgments are normative and some are not is that different frameworks are used to model the different types of judgments. Partly this is due to the fact that non-normative judgements are defined by what they are not, viz. those that can be explained by an underlying classical (Bayesian) belief state. One way of modeling non-normative judgments is via heuristics (see e.g., Gigerenzer et al, 2015) which may bear no relationship to classical probability computations. Another is to replace classical sample spaces with quantum vector (Hilbert) spaces, as done in quantum models of cognition (Busemeyer & Bruza, 2012), which again appear to have little relation to classical probability theory.

There would appear, therefore, to be a disconnect between the way cognitive states, and computations on them, are represented depending on whether one is dealing with normative or non-normative reasoning. This poses a challenge if we wish to understand the reasons why we might sometimes give normative judgments and sometimes not, or, for example, if we wish to understand how corrective feedback may improve performance.

What we want to do in this contribution is to introduce a way of thinking about non-normative judgments that makes them look as similar as possible to normative ones. This representation can be used in a variety of settings to study transitions between non-normative and normative behaviors. We will show that for non-normative judgments the notion that an underlying probability distribution 'does not exist' can be formalized by considering quasi-distributions, which are similar to standard probability distributions except that some elements may be negative. This gives us a way to think about smoothly transitioning from non-normative judgments, represented by quasi-distributions, to normative ones, where all elements of the distribution are positive and it may be interpreted as a classical probability distribution.

By itself this achieves little beyond expressing the problem of non-normative judgments in a different language, however we will argue that this representation provides a new way to understand the origin of non-normative judgments, and even a way of visualizing how learning can cause a transition to normative behavior. We will see that non-normative judgments can arise because of an misestimation of the correlation between events, and we will argue that this may occur when events are perceived to be highly similar. We test this in an experiment, looking at judgments about the joint occurrence of events with different degrees of similarity.

The rest of this contribution is structured as follows, first we provide a brief introduction to quasi-probability distributions, our aim being to show how they may be used to encode non-normative judgments. Next, we use this framework to develop a novel empirical prediction that a high degree of perceived similarity between features of an object can give rise to violations of independence and conjunction fallacies in probabilistic judgments. Then, we describe an experiment to test this prediction. We conclude with some possible avenues for further study.

Probability distributions and quasi-distributions

A common theme in experiments on probabilistic judgment is that participants are asked to make a set of judgments about the likelihood of some events, e.g. $p(A), p(B), p(A \cap C)$ etc, and the normative status of these judgments is assessed by proving that there either does or does not exist a probability distribution $p(A_i, B_j, C_k)$ such that all the measured judgments can be thought of as marginals of this joint distribution. For example, the conjunction fallacy, wherein participants judge $p(A \cap B) > p(A)$, is non-normative because it is impossible for participants to have a single joint probability distribution for A and B with this property.

Joint probability distributions

Establishing whether a set of probability or likelihood judgments is normative is therefore equivalent to the following:

Given some set of probabilities, $\mathcal{S} = \{p(A), ...p(A,B), ..., p(A,B,C), ...\}$ etc does there exist a joint distribution $p(A_i, B_j, C_k...)$ of which all elements of the set $\mathcal{S}$ may be considered as marginals?

If such a probability distribution exists, then the set of judgments $\mathcal{S}$ are normative, otherwise they are non-normative.

Some Examples

- This definition includes trivial cases, e.g. where $\sum_i p(A_i) \neq 1$. Such cases are obviously non-normative.
- A simple example is provided by the set $\mathcal{S} = \{p(A_i), p(B_j), p(C_k), ...\}$ where participants are only asked to make judgments about a single event. In this case we can easily find a joint distribution that has the single event probabilities as marginals, e.g. $p(A_i, B_j, C_k, ...) = p(A_i)p(B_j)p(C_k)...$ will work (there are many choices).

A Less Trivial Example Suppose we have three binary events A, B, C and we are given the joint probabilities, $\mathcal{S} = \{p(A_i, B_j), p(B_j, C_k), p(A_i, C_k)\}$. An important result is that it is *not* always possible to find a joint distribution $p(A_i, B_j, C_k)$ with these marginals. The conditions under which this is possible are when the Bell inequalities are satisfied (Fine, 1982).

Joint quasi-probability distributions

If a set of judgments $\mathcal{S}$ is not normative, then a probability distribution capturing these judgments does not exist. However there may still exist *some* function $q(A_i, B_j, C_k...)$ such that all the elements in the set $\mathcal{S}$ can be obtained by summing out the other variables in $q(...)$. This function $q(A_i, B_j, C_k...)$ will generally fail to be a probability distribution because it will not be non-negative.

Example Suppose $\mathcal{S} = \{p(A), p(B), p(A \cap B)\}$ for some binary valued features A, B and $p(A \cap B) > p(A)$. Clearly there is no probability distribution which can have $\mathcal{S}$ as its marginals. However a quasi-distribution with these properties may be given as:

$q(A,B) = p(A \cap B)$,
$q(A,\bar{B}) = p(A) - p(A \cap B)$,
$q(\bar{A},B) = p(B) - p(A \cap B)$,
$q(\bar{A},\bar{B}) = 1 - p(A) - p(B) + p(A \cap B)$.

Note that this has the desired marginals, e.g. $q(\bar{A},B) + q(\bar{A},\bar{B}) = 1 - p(A)$, but that $q(A,\bar{B}) < 0$.

The use of quasi-distributions in psychology to understand inconsistent judgments has been advocated before, most notably by de Barros, (e.g. de Barros, 2013). In physics there is a long history of trying to apply 'extended' probabilities to understand aspects of quantum theory (see e.g. Muckenheim, 1986). Their interpretation can be challenging (Halliwell & Yearsley, 2013) but here we shall avoid assigning any meaning to them and regard them simply as a computational tool.

So far all we have done is to express some classes of non-normative judgments in terms an object which is superficially similar to a joint probability distribution, but which fails to be one in some (rather drastic) way. Why is this useful? Well the usefulness of quasi-probability distributions lies in part in the fact that they smoothly capture the idea of transitioning between non-normative behavior (where one or more of the elements of the distribution is negative) to normative behavior, where all elements are non-negative. Suppose for example that we are performing an experiment where participants have to bet on the outcome of some gamble involving a conjunction. If they commit a conjunction fallacy in their reasoning, they may initially perform badly, but with corrective feedback they may revise their estimate of the probabilities of the outcomes. At some point their beliefs will change from non-normative to normative discontinuously, but in terms of quasi-distributions their their belief state may change smoothly as they learn.

Quasi-distributions also allow us to define a notion of distance from normative behavior, for example one could define the degree of non-normativity as,

$$\Delta = \sum_{i,j,k...} |q(A_i, B_j, C_k...)| - 1 \tag{1}$$

which is zero if $q(...)$ is a probability distribution and non-zero otherwise. For example, for the case above of a conjunction fallacy, $\Delta = 2|p(A,B) - p(A)|$, which is an appealing measure of the non-normativity.

One reasonable proposal would be to look at cases where a set of judgements $\mathcal{S}$ only just fails to be normative by this measure. One could then try to define a genuine probability distribution $p(...)$ and a new set of judgments $\mathcal{S}'$ which are 'close' to the real judgments $\mathcal{S}$ in the sense that $p(...)$ is close to $q(...)$. If there is a sense in which this is possible then one might regard the judgments $\mathcal{S}$ as *almost* normative, and perhaps attribute the discrepancy to some sort of response noise.

We will not pursue this further here. Instead we will make use of another advantage of quasi-distributions, which is that by expressing normative and non-normative behaviors in a similar language, they suggest ways to understand how non-normative behaviors may come about. We shall explore one such idea in the next section.

A Simple Proposal

In a typical conjunction fallacy type experiment, participants might be expected to have some information about the rate of occurrence of the features A and B, and be asked to guess the likelihood of the conjunction $p(A \cap B)$. It is important to realize that even given $p(A), p(B)$ there is no 'correct' answer to this question. Rather there are a range of possible allowed values, in other words, the marginal probabilities $p(A), p(B)$ under-specify the joint distribution. One extra piece of information is needed, one of the joint probabilities would do, as would some linear combination of these. One possibility is to consider the quantity,

$$S_{AB} = p(A \cap B) + p(\bar{A} \cap \bar{B}) \quad (2)$$

which is closely related to the correlation. The difference between a normative probability distribution and a non-normative quasi-distribution can be thought of, perhaps simplistically, as the difference between choosing a value for S_{AB} within or outside of the allowed range. This is important because decision makers armed only with $p(A), p(B)$ have no information about S_{AB}, and there is therefore a significant possibility that they may chose incorrectly. To put it another way, if decision makers make an incorrect guess for the correlation between A and B, this can lead to a non-normative set of judgments.

Now S_{AB} is a number which varies between 1 if the events always happen together, to 0 if the presence of one event implies the absence of the other and vice versa. One possibility is that decision makers simply pick a value for S_{AB} based on a uniform prior. Another possibility is that decision makers equate S_{AB} with a more primitive quantity such as the similarity between A and B. (We note in passing that the idea of similarity as essentially joint probability appears in accounts of similarity judgment based on quantum cognitive models (Pothos et al, 2015).)

This leads to an important prediction, which we will test below: In the absence of any information about joint occurrence, human decision makers will use features of events such as their similarity to construct a joint distribution. Manipulating these relationships in an experimental setting should lead to greater or lesser degrees of violations of independence for these events, and more generally to changes in the correlation between events. We describe an experiment to test these ideas in the next section.

An Experiment

Methods

58 undergraduate students from Vanderbilt University participated in the experiment online at a time of their choosing for course credit. Participants answered questions about three different novel categories, an animal, a natural object and a human made object, adapted from previous work on causal reasoning (Rehder, 2014) . Each object had three binary features (A, B, and C). For each feature participants were told that 'most' members of the category had a high value for that feature, while 'a few' members of the category had a low value for the feature. Participants were not told about any relationships between the features. For example, in the Kehoe Ant category, A = Blood iron level (high or low amount), B = Immune system activation level (hyperactive or suppressed), and C = Blood thickness (thick or thin).

After this, participants answered a number of questions where they were told that a new member of the category had been discovered, and were asked to indicate how likely they thought it was that the new object had various features. There were three question types: (1) how likely it was that the object had a particular feature, e.g. blood high in iron sulphate, (2) how likely it was that the object had a combination of features, e.g. blood high in iron sulphate and an immune system that is hyperactive, and (3) a conditional, e.g. a hyperactive immune system given that a previous test had established a high level of iron sulphate in the blood. Participants were asked about all possible conjunctions of events, but only about conditionals where one feature was conditioned on the presence of a low value for another feature. The reason for this was to reduce the overall number of questions in the experiment, particularly since our expectation was that features would be positively correlated, which would be likely to lead to floor or ceiling effects for the other possible conditionals.

The responses were either requested as whole numbers between 0 and 100, or as points on a 9 point Likert scale. The response format for all questions concerning a given category were the same, and participants were randomly assigned either the whole number or the Likert response options for each category. After completing the likelihood judgment questions for each category, participants were asked to rate the similarity between the feature types on a 7 point Likert scale. The order in which the features appeared in the similarity question (e.g. how similar is feature 1 to feature 2) was randomized between participants for every judgment, however there were no significant order effects in the similarity judgments.

After finishing the main part of the experiment, participants completed an extended version of the Cognitive Reflection Test (CRT, Frederick, 2005), but there was no significant effect of CRT and it will not be discussed further here.

Results

We first wanted to examine whether we had conjunction fallacies and violations of independence in this data set. For each pair of likelihood judgments, e.g. $\{A, A \cap B\}$ or $\{A, A|B\}$ we can perform a paired samples t-test to assess the presence of these effects, however this procedure would generate a substantial volume of test statistics without giving much insight. Instead we will plot the relevant likelihoods, and quote some representative statistics. In this contribution, we only report Bayesian statistical tests that were performed using JASP (JASP team, 2016). In particular we report Bayes factors for the alternative versus the null hypothesis, so that values > 1 indicate evidence for the alternative hypothesis.

We begin by assessing the conjunctions. For each pair $\{A, A \cap B\}$ we plotted the average values across participants of the single event and the conjunction. It is useful to split these pairs up into four different types, depending on whether each of the events has high or low individual probability. The results are shown in Fig 1, and we have separated out data that comes from responses using whole numbers and from the Likert scale. Points that lie above the diagonal correspond to conjunction fallacies. The first thing to note is that there are three obvious clusterings of data points. We see straight away that pairs of the form $\{A, A \cap \bar{B}\}$ behave as expected, there are no conjunction fallacies. Equally the pairs $\{A, A \cap B\}$ do not display robust conjunction fallacies (All Bayes factors for t-tests < 1), although these data points are slightly odd in another way, which we will return to shortly. The pairs which do display conjunction fallacies are $\{\bar{A}, \bar{A} \cap B\}$ and $\{\bar{A}, \bar{A} \cap \bar{B}\}$.

The pairs of the form $\{\bar{A}, \bar{A} \cap B\}$ display robust conjunction fallacies. For the whole number responses 14 out of 18 of the Bayes factors for t-tests are > 3, and 10 out of 18 are > 10. For the Likert responses 16 out of 18 of the Bayes factors for t-tests are > 3, and 11 out of 18 are > 10. These are the pairs for which conjunction fallacies are typically expected, with one likely and one unlikely event. In contrast the presence of conjunction fallacies in the pairs of the form $\{\bar{A}, \bar{A} \cap \bar{B}\}$ are less expected. For the whole number responses none of the Bayes factors for t-tests are > 3, but for the Likert responses 7 out of 18 of the Bayes factors for t-tests are > 3, and 5 out of 18 are > 10. We will return to why this may be so later.

Overall then, we have good evidence for conjunction fallacies in some of these judgments, for both response types. Note also that there do not appear to be large systematic differences between the data obtained from different response modes, which is reassuring.

Next we check for violations of independence. Note that these violations are not necessarily non-normative, since no information about the relationship between features was given to participants. However systematic violations of independence would still be a surprising finding. We proceed as for the conjunctions, plotting the pairs $\{A, A|\bar{B}\}$ and $\{\bar{A}, \bar{A}|\bar{B}\}$ separately and also separating out whole number and Likert responses. The results are shown in Fig 2. Independence would be indicated by data points lying on the diagonal.

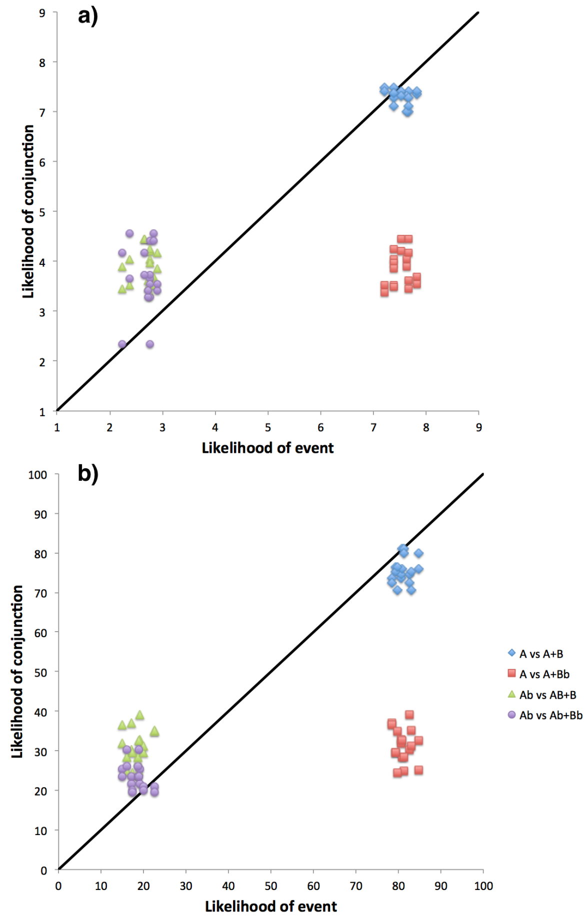

Figure 1: Plots of likelihood judgments for conjunctions against single constituent events. Data points above the diagonal indicate conjunction fallacies. a) Likert scale responses. b) Whole number responses.

For the pairs of the form $\{A, A|\bar{B}\}$ all points appear to lie below the diagonal, and this is confirmed by Bayesian t-tests. For the whole number responses and for the Likert responses all Bayes Factors are > 10.

For the pairs of the form $\{\bar{A}, \bar{A}|\bar{B}\}$ all points appear to lie above the diagonal, and this is confirmed by Bayesian t-tests. For the whole number responses 13 out of 18 Bayes Factors are > 3, and 7 out of 18 are > 10. For the Likert responses we also have 13 out of 18 Bayes Factors > 3, and 7 out of 18 > 10. Again overall there is good evidence for violations of independence in this data.

More specifically, the conjunctions and the conditionals point to similar behavior - namely participants appear to believe that there is strong correlation between the different features, such that "high" or "low" values of these features are likely to occur together.

Now we turn to the question of whether the perceived similarity between features mediates the correlations and violations of independence.

We begin with the correlation, defined for each pair of features A, B as $p(A \cap B) + p(\bar{A} \cap \bar{B}) - p(A \cap \bar{B}) - p(\bar{A} \cap B)$. We ran a Bayesian ANOVA with the perceived similarity as the

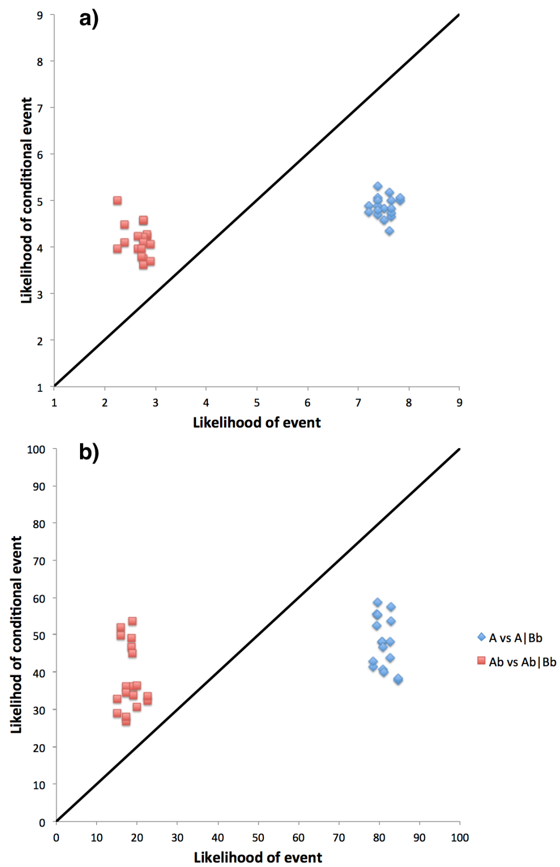

Figure 2: Plots of likelihood judgments for conditionals against the conditioned events. Data points on the diagonal indicate independence. Data points off the diagonal indicate violations of independence. a) Likert scale responses. b) Whole number responses.

independent variable. Note that we collapse across features and scenarios here, since we have similarity data for each individual feature pair.

For the whole number responses the results of the Bayesian ANOVA show that a model including perceived similarity is preferred over the null model ($BF_{10} > 10^7$). For the Likert responses the results of the Bayesian ANOVA also show that a model including perceived similarity is preferred over the null model ($BF_{10} > 10^3$). Analysis of effects for both ANOVAs are given in Table 1.

We plot in Fig 3a below the correlation as a function of the similarity. Since we saw in the analysis of the conjunctions and conditionals there were no obvious differences between the response types we have converted the Likert scale responses to numbers in the range 0-100 and plotted them on the same axis. This lets us establish that the same qualitative pattern holds for both response types, namely there is some apparent decrease in correlation for very small similarity ratings, but then a robust increase in correlation with increasing perceived similarity. The reason for the decrease in correlation for small similarity ratings is unclear, although it is worth noting that the number of participants who gave similarity ratings from 1-3 is small (7-11) compared with the number of participants who gave higher similarity ratings (40-66). In addition, for the lowest similarity rating a higher than expected proportion of participants giving this rating (5 out of 8 for the Likert scale and 6 out of 9 for the whole numbers) had the highest possible CRT score. This is significant because if two events, each of which has an individual probability of 0.8, are independent, then the expected correlation is 0.32, which is in fact close to the observed value for a similarity rating of 1 in the whole numbers condition.

Next we analyze the violations of independence. We compute a violation 'score' which is just the sum of the absolute value of the difference between the conditional and the single event, e.g. $|p(A|B) - p(A)|$ for all the conditionals we measured. Again we ran a Bayesian ANOVA, with perceived similarity as the independent variable.

For the whole number responses, the results of the Bayesian ANOVA show that a model including perceived similarity is preferred over the null model ($BF_{10} \sim$ "∞"). For the Likert responses the results of the Bayesian ANOVA show that a model including perceived similarity is again preferred over the null model ($BF_{10} > 10^{10}$). Analysis of effects for both ANOVAs are given in Table 2.

In Fig 3b we plot the violation of independence score as a function of perceived similarity. Again we transform the values for the Likert scale responses allowing us to plot them on the same axes. The pattern is qualitatively similar to that for the correlation; a general trend towards larger violations of independence for higher values of the perceived similarity.

Table 1: Analysis of effects for Bayesian ANOVA of Correlation

Whole Numbers

Effect	$p(incl)$	$p(incl\|data)$	$BF_{Inclusion}$
Similarity	0.500	1.000	3.01×10^7

Likert Scale

Effect	$p(incl)$	$p(incl\|data)$	$BF_{Inclusion}$
Similarity	0.500	0.999	1.58×10^3

Table 2: Analysis of effects for Bayesian ANOVA of Violations of Independence

Whole Numbers

Effect	$p(incl)$	$p(incl\|data)$	$BF_{Inclusion}$
Similarity	0.500	1.000	"∞"

Likert Scale

Effect	$p(incl)$	$p(incl\|data)$	$BF_{Inclusion}$
Similarity	0.500	1.000	3.08×10^{10}

Overall the data provide strong support for our proposal that perceived similarity mediates perceptions of correlation and violations of independence in probabilistic judgments. It is also worth noting that there are strong positive correlations between the correlation function and violations of independence, (Pearson's rho = 0.481, $BF_{10} > 10^{14}$ for whole number responses, Kendall's tau = 0.345, $BF_{10} > 10^{13}$ for the Likert responses.)

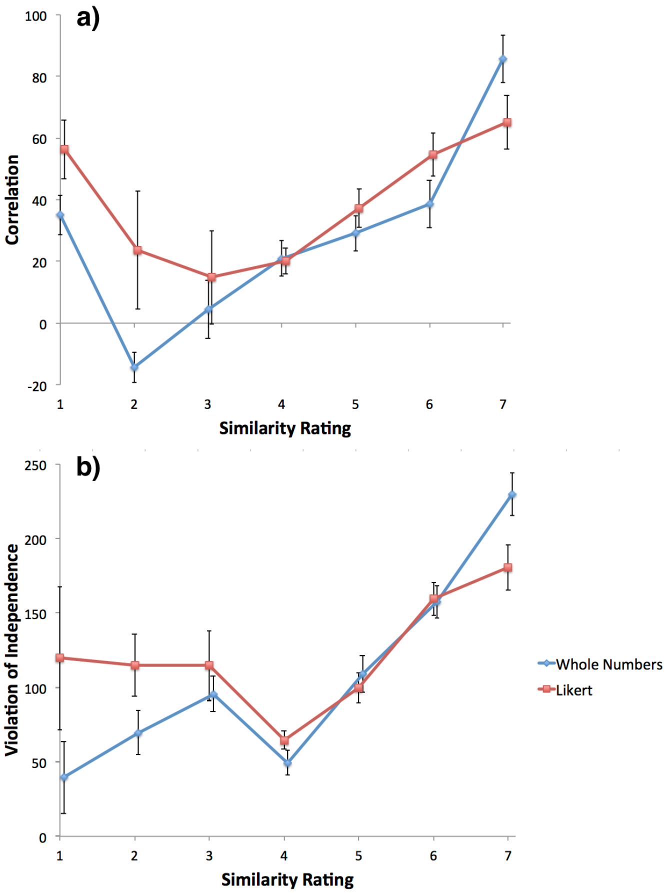

Figure 3: Plots of the correlation and degree of violation of independence against the perceived similarity. a) Correlation as a function of perceived similarity for the whole number responses (blue line) and the Likert scale responses (red line.) b) Violations of independence as a function of perceived similarity for the whole number responses (blue line) and the Likert scale responses (red line.)

Conclusions and Future Directions

We have shown that quasi-probability distributions can be used to encode certain sets of probabilistic judgments which are non-normative, in the sense that they cannot be regarded as marginals of a joint probability distribution. Quasi-distributions generalize regular probability distributions in that they can have negative elements. By themselves these do not provide any great insight into non-normative behavior, but the fact that one can define analogues of properties such as correlations for quasi-distibutions lets us examine the ways in which sets of judgments fail to be normative, and perhaps suggest some possible reasons why. We proposed that in the absence of information about joint occurrence, human decision makers might use properties such as the similarity between features to set the correlation, which we demonstrated experimentally. Similarity does seem to mediate correlation and violations of independence. We also showed that these results are largely independent of the response format.

These findings are particularly significant for attempts to assess the normative status of human causal inference using stimuli of this nature (e.g. Rehder, 2014). In these experiments, participants are given extra information about the features in the form of causal relationships between them. Judgments about correlations in this case could then reasonably be interpreted as meaning participants believe the presence of one feature *caused* another. This work suggests that care is needed when interpreting these studies - participants may believe that features are correlated even in the absence of causal relationships, which may lead to overestimation of perceived causality. Future work should explore this possibility.

Finally, the results of this study suggest quasi-distributions may be a valuable way of thinking about non-normative reasoning, and we are hopeful that this approach may be used fruitfully in other areas. One important task is the development of a learning model which works directly with quasi-distributions. This could help us understand how people learn to avoid committing probabilistic fallacies (Nilsson, 2008).

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