

A Computational Model of the Role of Attention in Subitizing and Enumeration

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Abstract

Recent studies in the perception of numerosity have indicated that subitizing (the rapid and accurate enumeration of small quantities) requires *attention*. We present a novel computational model of enumeration in which attention unifies distinct processes of numerosity approximation, subitizing, and explicit counting. We demonstrate how this model accounts for both the reaction time results from the subitizing literature and the effects of attentional load on subitizing accuracy.

Keywords: attention; subitizing; enumeration; perception of numerosity; counting; inattentional blindness

Introduction

The perception of *numerosity* is one of the core faculties underlying much of the human ability to represent, reason, and communicate about number (Dehaene, 2011), and the precision and accuracy of this “number sense” is highly dependent on the amount of *attention* devoted to it. Imagine that you are asked to report the size of a small crowd gathered around a performer in a public park. Glancing at the crowd might suffice to obtain a rough figure (e.g., about thirty), but obtaining an exact number with high confidence would require looking at individual people and counting them. Converging lines of evidence suggest that numerical cognition is supported by two types of representations of numerosity (Feigenson, Dehaene, & Spelke, 2004). The first is an *approximate* representation that can be quickly perceived through estimation. The second is a precise representation, associated with linguistic representations of number (Gelman & Butterworth, 2005) that are usually generated through a slower process of explicitly fixating on and counting individual objects.

Yet, a simple dual-system account of numerosity judgment is incomplete. Since the 19th century, scientists have observed that the enumeration of small quantities of objects (under five) was both rapid and accurate (Jevons, 1871). This phenomenon is called *subitizing* (Kaufman, Lord, Reese, & Volkman, 1949). Within the subitizing range, each additional object requires only 40–100 ms more time to enumerate on average, whereas in the post-subitizing range, each additional object requires 250–350 ms to enumerate (Trick & Pylyshyn, 1994). Previous work has proposed that subitizing is the result of a parallel and pre-attentive process (Trick & Pylyshyn, 1994; Mandler & Shebo, 1982) that transitions into slower, serial counting.

In contrast with this account, recent studies provide a body of evidence demonstrating that attentional manipulations, such as attentional blink or increased attentional load, adversely affect subitizing performance (Railo, Koivisto,

Revonsuo, & Hannula, 2008; Olivers & Watson, 2008; Egeth, Leonard, & Palomares, 2008; Vetter, Butterworth, & Bahrami, 2008). Therefore, subitizing has an attentional component, and an account of the processes underlying subitizing must explain both (a) why subitizing is significantly more rapid than explicit counting and (b) why it requires attention.

We claim that numerosity judgments of all types are subject to constraints on attention and that attention flexibly integrates the results of multiple number-processing capacities. Both classical and newer results in the subitizing literature can be explained by combining a capacity-limited, object-based view of attention with a capacity for approximation that operates independently of attention. In this paper, we present a computational model of enumeration in which attention unifies distinct processes of numerosity approximation, subitizing, and explicit counting. We begin by introducing a computational cognitive modeling system, ARCADIA (Bridewell & Bello, 2016b), in which attention features centrally. We then proceed to present a computational account of enumeration, proposing a model of rapid, serial subitizing in which the numerosity of objects encoded in visual short-term memory can be quickly established. We then demonstrate how this model of subitizing accounts for both the reaction time and the effects of attentional load. Finally, we discuss the various processes involved in the perception of numerosity and the role of attention in enabling and integrating them.

ARCADIA’s Architectural Features

ARCADIA provides a system in which *attention* is the primary organizing mechanism for perception, cognition, and action. In this section, we provide a brief overview of the key concepts necessary to understand ARCADIA models and point to a more detailed introduction by Bridewell and Bello (2016b). Each ARCADIA model consists of a set of *components*, which carry out all the processing for the model, that can read from or write to *accessible content*, a temporary buffer where components share their output on each cycle. Whereas representations inside of individual components can take arbitrary form, every component is designed to read from and express its results in a common representation called the *interlingua*. This shared language pulls together data in different formats so that they can be exchanged among components.

ARCADIA operates in discrete, cognitive cycles, each corresponding to 25 ms intervals (Bridewell & Bello, 2016a). During each cycle, an *attentional strategy* selects an element

out of accessible content to be the *focus of attention*. Some components can respond to the focus of attention, while others operate independently of the focus. In this way, ARCADIA has natural resources for distinguishing processes that require attention from those that do not.

Model of Enumeration

In this section, we present an ARCADIA model of numerosity judgment that includes both estimation and counting processes. The core processes within this model are illustrated in Figure 1. We model counting such that differences in enumeration speed result from the attentional and temporal constraints necessitated by maintaining count information in different memory stores. The model proposes that a count of the first set of seen objects can be calculated from visual short-term memory (vSTM) and represents the subitized count. After this, an explicit, symbolic representation of the current count is maintained that requires rehearsal via subvocalization (i.e., inner speech) supported by a phonological loop.

The connection between subvocalization and slow counting has been previously established (Dehaene, 2011), and evidence has shown that articulatory suppression adversely affects counting performance (Logie & Baddeley, 1987). Likewise, vSTM capacity-limits have been proposed as an explanation of subitizing (Railo et al., 2008), and there is evidence that vSTM load affects subitizing performance (Cutini & Bonato, 2012).

Visual Processing

For the sake of brevity, our discussion of these components is abridged. A more detailed walkthrough of visual processing mechanisms in ARCADIA can be found in Bridewell and Bello (2016a).

Early Vision: At the first stage of visual processing, image segmentation produces a set of segments, corresponding to hypotheses about object locations that we call proto-objects (Rensink, O’Regan, & Clark, 1997). [Step 1].

Fixation Generation: Highlighter components produce candidate fixations on either individual proto-objects or a group of proto-objects. When a candidate fixation on a single proto-object becomes the focus of attention, an object-binding process begins [Step 2a]. If a candidate fixation for a group of proto-objects becomes the focus of attention, an estimation process is triggered [Step 2b].

Object Binding and Storage (vSTM): When a candidate fixation containing a single proto-object is focused on, a (visual) feature-binding process occurs, creating an object representation containing size, shape, color, and location information. This object representation can in turn be focused on, which updates vSTM. The ARCADIA model of vSTM has a maximum capacity of four objects (Bridewell & Bello, 2016b).

Task Knowledge (Visual Cues): Task knowledge includes object property information used as visual cues to identify and distinguish between different types of task-relevant objects. This includes size and shape information about objects

that should be enumerated and mask objects that represent the end of a trial.

Enumeration Components

Various components are responsible for subitizing and subvocal counting, estimation, and then merging these results into a single numerosity report.

Subitized Numerosity: When vSTM is full, or a mask object is detected, or when there are no more uncounted objects, then the number of objects to be enumerated in vSTM is returned. Note, this may be based on a subset of the objects in vSTM. Evidence supports the notion that people are able to selectively enumerate objects in vSTM based on various object properties (Chesney & Haladjian, 2011).

Lexical Count: For precise numerosity representations to be remembered, they first must be converted into a lexicalized form (e.g., “one,” “four,” “eight”). This happens when a subitized numerosity representation is focused on [Step 3]. When a lexicalized count representation already exists in working memory, and a new object is focused on, the next lexicalized number in the count sequence is returned [Step 5]. The presence of lexicalized number representations triggers subvocalization in the phonological loop.

Phonological Loop: The phonological loop is implemented with a component that generates a series of subvocalization actions over multiple cycles, ensuring that the number of sequential subvocalization actions corresponds to a model of subvocalization time. Currently, the number of ARCADIA cycles required to subvocalize a word is calculated based on the approximation from Huss and Byrne (2003): 150 ms per syllable and one syllable per three characters in a lexeme. For example, “seventeen” would have three estimated syllables for a total subvocalization time of 450 ms.

Approximate Number System (ANS): The ANS responds when the focus is a fixation on a group of proto-objects, by producing a noisy number-sense representation [Step 4]. The result is a normal distribution with a mean of n_e (the number of proto-objects in the group fixation region) and standard deviation of $w \cdot n_e$, where w is the model’s Weber fraction parameter (Halberda & Feigenson, 2008).

Numerosity Reporter: The numerosity reporter is responsible for merging both the results from the ANS and the serial count into a single numerosity judgment. If time allows for an explicit count to be fully generated, the explicit count is recorded. Otherwise, an educated guess is made:

$$Guess(n_c, n_e, w) = n_c + sample(\mathcal{N}(n_e - n_c, \sqrt{w \cdot (n_e - n_c)}))$$

where n_c denotes the number of explicitly counted objects. This reflects the basic phenomenon that enumeration error decreases gradually in conjunction with the number of items able to be explicitly counted (e.g., Mandler & Shebo, 1982; Railo et al., 2008). Whether this can be fully explained by approximation on uncounted fixation candidates or approximate mathematical operations on partial results (e.g., Gallistel & Gelman, 2000) is a topic for further investigation.

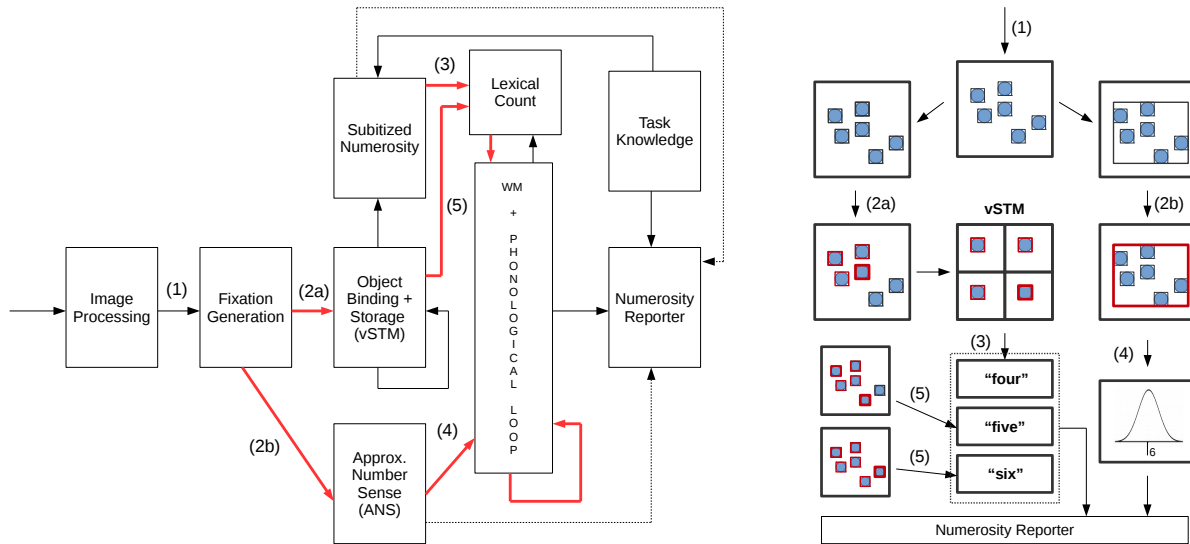


Figure 1: Information flow within the ARCADIA model of enumeration. Red lines indicate information flow that requires focus of attention.

Modeling Rapid vs. Slow Counting

In this section, we demonstrate the application of the ARCADIA numerosity perception model to a basic enumeration task (the enumeration of one to eight objects with no effective time limit). Our goals in examining this base case are to investigate and account for the origins of the bilinear reaction time curve found in the subitizing literature. To evaluate the model, we generated 40 videos, each containing a randomized, irregular pattern of one to eight (non-overlapping) circles. There were five videos for each number of circles, and the model was run 10 times for each video.

Attentional Strategy

Attentional strategies in ARCADIA determine which element from accessible content becomes the focus of attention on a given cognitive cycle. The attentional strategy for the enumeration model embodies two constraints and prioritizes selection to jointly satisfy them. First, each task-relevant object must be counted. As such, after a new candidate proto-object is focused on, priority is given to processing that updates the count. Second, objects must not be counted more than once. Candidate proto-object fixations are ordered in a left-to-right manner, and the position of the last counted object is tracked.

Results

Figure 2 provides a plot of the simulated reaction times from the ARCADIA model of counting (25 ms per cycle) with human subject results from Trick and colleagues (1996). The simulated RTs from the model are consistent with human subjects from the 22 year-old group, $r^2 = .990, p < .001$.

The model predicts that enumerating a single object requires at least 375 ms (or 15 ARCADIA cycles). The first seven cycles are needed to generate a lexicalized representation of numerosity for report. The first cycle is needed for

image processing to occur and fixation candidates to be generated [Step 1]. The first fixation candidate is focused on in the second cycle [Step 2a]. The third cycle is required for object binding, and the fourth cycle is required to encode the object representation into vSTM. The fifth cycle is required for the subitized numerosity process to determine that there are no more unvisited candidate fixations. Finally, the sixth and seventh cycles are required for the subitized numerosity process to produce an object count from vSTM [Step 3] and for this numerosity representation to be converted into lexicalized form. The remaining eight cycles are required for subvocalization and generation of the final numerosity report.

When vSTM is below capacity (not filled to its four item limit) only 50 additional milliseconds are required for each additional item to be enumerated, which is consistent with the 40–100ms per additional item result from the subitizing literature (Trick & Pylyshyn, 1994). During subvocalized counting each new enumerated object necessitates updating the last enumerated point in working memory (to keep track of which points were counted) and an explicit subvocalization of the updated count. This additional attentional requirement adds roughly 250–350 ms per item (75 ms for object binding and inhibition updating and 175–275 ms for subvocalization and number report). Simulated RTs from the ARCADIA model are more consistent with previous human studies that the Peterson and Simon (2000) ACT-R model of subitizing (SUBIT-R), which predicts enumerating one to two objects as taking roughly 200 ms and over 1000 ms for four objects.

Modeling Enumeration: Attentional Effects

As a second test of the model, we replicate the results from Railo and colleagues (2008). In that study, the authors used a paradigm originally applied to study inattentional blindness (Rock, Linnett, Grant, & Mack, 1992). Subjects had two

potential tasks: (1) report which line of a centrally located cross is longer and (2) report the number of dots clustered in a quadrant outside the central cross. Videos consisted of a series of trials in which the cross appeared, with a critical trial in which a peripheral dot cluster appeared for the first time (and subjects were unaware of the enumeration task).

There were three experimental conditions (for more detail see Bridewell and Bello, 2016a). First, the *inattention* condition consisted of the critical trial. Subjects were asked first to report the results from the length comparison task. After the third, critical trial, subjects were asked whether they noticed any dots and, if so, to report their quantity. Next, the *divided-attention* condition consisted of trials in which subjects were asked to perform both tasks. Finally, the *full-attention* condition consisted of trials in which subjects were told to ignore the length comparison task and focus only on enumeration. In the inattention condition, subjects who were not inattentionally blind to the dots (~80%) had enumeration accuracy close to 100% for up to two dots, after which accuracy dropped to under 25%. In the divided attention condition, enumeration accuracy was at or near 100% for one and two dots, after which accuracy more gradually declined. In the full attention condition, enumeration accuracy was at or near 100% for up to three dots before beginning a decline. Because the attentional manipulation is the result of a dual task, additional components are necessary to allow the model to produce results for both, which we outline below.

Model Configuration

We generated 36 videos to serve as stimuli for our simulation of the task environment with six videos for each number of peripheral dots. Each video consisted of a fixation cross presented for 1500 ms followed by a 200 ms stimulus interval with a centrally located cross and peripheral dot cluster. After the stimulus interval in each video, a mask was displayed for 500 ms. The model was run on each video 20 times.

Additional Components Bridewell and Bello (2016a) present a computational model of inattentional blindness applied to similar stimuli. The current stimuli use the same paradigm, and as such many of the components from that early model are reused.¹

Attentional Strategies As in the previous enumeration model, ARCADIA follows a left-to-right prioritization of candidate fixations for enumeration. However, in the inattention and divided attention conditions, peripheral fixations are inhibited until a length comparison result is encoded in working memory (ensuring enumeration processing occurs after the primary task is complete). After this, in the divided attention condition, group fixations are given prece-

¹Specifically, the previous model possessed six components used to achieve the primary cross-dimension comparison task: the *object height reporter* and *object length reporter*; the *length comparator*, which compared the results from the object height/length reporters; the *comparison recorder*; and the *center highlighter* and *orientation inhibitor*, which generated a fixation on the center proto-object and inhibition on peripheral proto-objects, respectively.

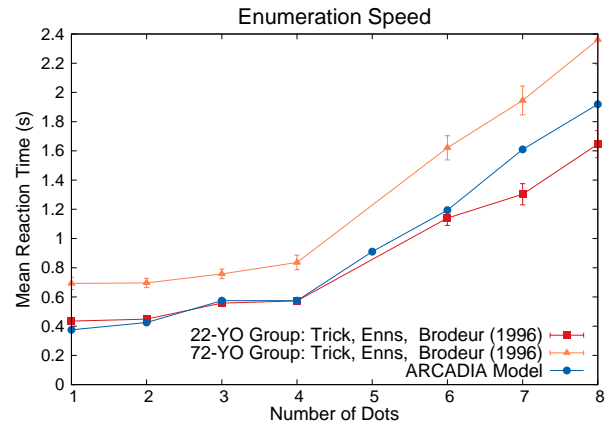


Figure 2: Simulated RT for ARCADIA enumeration model compared with human data from Trick and colleagues (1996).

dence over individual fixations (until an ANS result is memorized), whereas in the inattention case, group fixations are not prioritized (reflecting lack of awareness of the dot clusters and the time limited enumeration task). In the full attention condition, peripheral inhibition is absent, length comparison task elements are not prioritized, and group fixations are prioritized over individual fixations.

Results

Bridewell and Bello (2016a) analyzed focus traces generated by ARCADIA to demonstrate the precise effect attentional and temporal constraints had on the processing of inattentional blindness stimuli. In Figure 3, we present similar focus traces for each experimental condition in the dual task. Because the stimulus interval for the dual task was 200 ms, only eight ARCADIA cycles were available for completing both tasks. The primary (length comparison) task required four cycles to accomplish, leaving only four cycles for any enumeration or secondary processing. The amount of enumeration processing in these four cycles (or eight in the case of the full attention condition) influences how accurate the numerosity judgment could be.

In the inattention condition, after the primary task is accomplished, there is time to fixate on and encode two dots into vSTM. However, these fixations are incidental, and because the attentional strategy in the inattentional case does not prioritize focus on any group fixations, there are no resulting estimation results from the ANS. As such, there is no basis for an educated guess. If there were only one or two dots, then a correct numerosity judgment would be made. Otherwise, the system would generate an incorrect report. In contrast, the system generates an estimate when the attentional strategy is configured for the divided attention condition. However, there is time to generate and memorize the ANS estimate only and not to begin a serial estimation process. Therefore, accuracy begins to drop after one item, which corresponds to the increased noise associated with the ANS judgment (and subsequent guess inaccuracy). Finally, in the full attention condition, estimation can occur in the time that would have

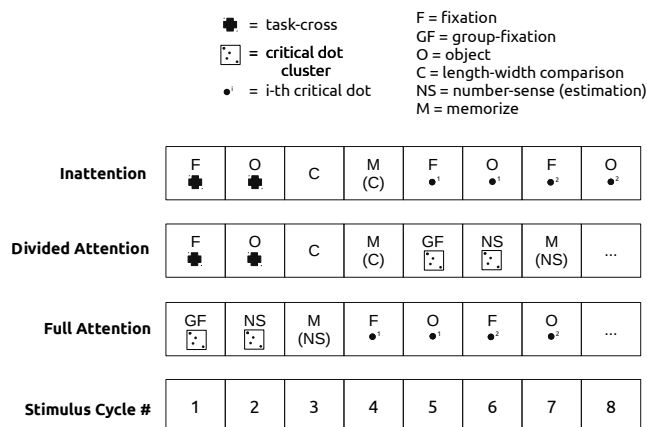


Figure 3: Focus traces from ARCADIA model of Railo et al. (2008) dual-task in different attentional conditions.

otherwise been required for the primary task. This provides time for two dots to be fully encoded in vSTM and explicitly enumerated. As such, performance does not begin to significantly decline until there are four dots or higher.²

The resulting performance curve can be found in Figure 4. A Weber fraction w of 0.13 was used, which is consistent with the range observed in normal adults (Halberda & Feigenson, 2008). Fischer’s exact tests showed that accuracy in the full attention condition was significantly higher than in the divided attention condition for dot numbers of two ($p = .002$), three ($p < .001$), four ($p < .001$), five ($p < .001$), and six ($p < .001$). Qualitatively, this matches the previously described attentional effects from the human subjects, with exception of the two dot results. Railo and colleagues did not find significant differences in performance for three or five dots, but ascribed this to potential perceptual difficulties in their stimuli. Our model may underestimate accuracy for two dots in the absence of other sources of numerosity information (i.e., pattern recognition, which we discuss in the next section).

Discussion

The boundary between the subitizing range and the post-subitizing range is commonly found to be four objects (Atkinson, Campbell, & Francis, 1976). Trick and Pylyshyn (1994) proposed that the four-object subitizing range (and the rapid enumeration within this range) emerges out of the limited capacity of “pre-attentive” individuation mechanisms. This “pre-attentive” characterization of subitizing stands in contrast to the serial, attention-bound enumeration mechanisms of explicit counting. However, in light of studies showing attentional effects within the subitizing range (e.g., Railo et al., 2008), the pre-attentive characterization must be re-evaluated.

The results from Railo and colleagues (2008) support two key points about the role of attention in enumeration. First, subitizing requires serial focus on individual objects. Other-

²When there are three dots, only one dot goes uncounted, which is approximated with high accuracy.

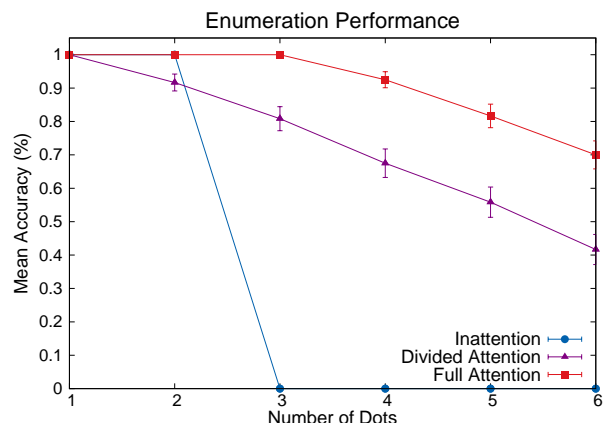


Figure 4: Mean model accuracy in different attentional conditions.

wise, there would be no significant performance differences between the divided and full attention conditions. In other words, subitizing (at least for arbitrary, irregular patterns) is unusually rapid, serial counting. The results also support the notion that even enumeration processes like the ANS have an attentional dependence, as subject performance in the divided and full attention conditions declined more gradually than in the inattention condition. In the inattention case, subjects attended incidentally to one or two dots, but in the absence of an intention to enumerate, they likely did not engage in approximation. As such, performance did not gradually decline. Attention, therefore, is a necessary feature of any unified account of numerosity perception.

The contrast between parallel and serial processes in enumeration may better be characterized by a distinction between a *weak* and *strong* sense of attentional involvement (rather than “pre-attentive” vs. “attentive”). Enumeration, in general, requires an intention to report on the absolute or relative quantity of objects in a visual scene. Therefore, there is at least the need to attend to the results of a parallel mechanism of numerosity judgment (the weak sense). In contrast, precise enumeration via counting requires attentional focus on each individual object to be enumerated (the strong sense).³

Other Subitizing Processes Regular and common patterns of objects (e.g., such as patterns found on dice) enable rapid and accurate numerosity beyond four objects (Mandler & Shebo, 1982), suggesting that pattern recognition may play a role in subitizing performance for certain spatial arrangements of objects. We view the reported model of subitizing as a complementary rather than competing account to pattern-recognition based ones (Peterson & Simon, 2000). A pattern-recognition component could be subsumed into the ARCADIA model as an alternate (and attentionally prioritized) number sensor that responds to focus on groups of proto-objects. This addition would enable us to make and model the following prediction: regular patterns such as those

³Our usage of the terms serial and parallel in this paper align with this strong and weak sense of attentional dependency, respectively.

presented by Mandler and Shebo (1982) would show minimal performance differences between the divided-attention and full-attention conditions in the dual task from Railo and colleagues (as compared to the irregular patterns used in the original study and in this paper).

As such, we view the perception of numerosity as a potentially four-part phenomenon. Estimation provides a rapid, parallel, but imprecise source of numerosity information, whereas subvocal counting provides a serial, slow, but accurate enumeration procedure. Subitized counting and pattern recognition provide both a serial and parallel mechanism, respectively, to achieve rapid and accurate enumeration.

To summarize, we have presented a novel computational model of numerosity perception in which attention unifies processes of subitizing, subvocal counting, and estimation. *Attention* is the glue that enables and binds these separate numerosity faculties together. The limits of attention-bound processes such as object-binding and subvocalization determine how quickly subjects can report numerosity judgments. Likewise, serial attentional focus to individual objects and the need for explicit attention to estimation is necessary to account for the accuracy of enumeration in dual-task settings.

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References

- Atkinson, J., Campbell, F. W., & Francis, M. R. (1976). The magic number 4 ± 0 : A new look at visual numerosity judgements. *Perception*, *5*, 327–334.
- Bridewell, W., & Bello, P. F. (2016a). Inattention blindness in a coupled perceptual–cognitive system. In *Proceedings of the thirty-eighth annual conference of the cognitive science society* (pp. 2573–2578). Philadelphia, USA.
- Bridewell, W., & Bello, P. F. (2016b). A theory of attention for cognitive systems. In *Proceedings of the fourth annual conference on advances in cognitive systems* (pp. 1–16). Evanston, USA.
- Chesney, D. L., & Haladjian, H. H. (2011). Evidence for a shared mechanism used in multiple-object tracking and subitizing. *Attention, Perception, & Psychophysics*, *73*, 2457–2480.
- Cutini, S., & Bonato, M. (2012). Subitizing and visual short-term memory in human and non-human species: A common shared system? *Frontiers in Psychology*, *3*, 129–133.
- Dehaene, S. (2011). *The number sense: How the mind creates mathematics*. New York, NY: Oxford University Press.
- Egeth, H. E., Leonard, C. J., & Palomares, M. (2008). The role of attention in subitizing: Is the magical number 1? *Visual Cognition*, *16*, 463–473.
- Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences*, *8*, 307–314.
- Gallistel, C. R., & Gelman, R. (2000). Non-verbal numerical cognition: From reals to integers. *Trends in Cognitive Sciences*, *4*, 59–65.
- Gelman, R., & Butterworth, B. (2005). Number and language: How are they related? *Trends in Cognitive Sciences*, *9*, 6–10.
- Halberda, J., & Feigenson, L. (2008). Developmental change in the acuity of the “number sense”: The approximate number system in 3-, 4-, 5-, and 6-year-olds and adults. *Developmental Psychology*, *44*, 1457.
- Huss, D., & Byrne, M. (2003). An ACT-R/PM model of the articulatory loop. In *Proceedings of the fifth international conference on cognitive modeling* (pp. 135–140). Boston, USA.
- Jevons, W. S. (1871). The power of numerical discrimination. *Nature*, *3*, 281–282.
- Kaufman, E. L., Lord, M. W., Reese, T. W., & Volkman, J. (1949). The discrimination of visual number. *The American Journal of Psychology*, *62*, 498–525.
- Logie, R. H., & Baddeley, A. D. (1987). Cognitive processes in counting. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *13*, 310–326.
- Mandler, G., & Shebo, B. J. (1982). Subitizing: An analysis of its component processes. *Journal of Experimental Psychology: General*, *111*, 1–22.
- Olivers, C. N., & Watson, D. G. (2008). Subitizing requires attention. *Visual Cognition*, *16*, 439–462.
- Peterson, S. A., & Simon, T. J. (2000). Computational evidence for the subitizing phenomenon as an emergent property of the human cognitive architecture. *Cognitive Science*, *24*, 93–122.
- Railo, H., Koivisto, M., Revonsuo, A., & Hannula, M. M. (2008). The role of attention in subitizing. *Cognition*, *107*, 82–104.
- Rensink, R. A., O’Regan, J. K., & Clark, J. J. (1997). To see or not to see: The need for attention to perceive changes in scenes. *Psychological Science*, *8*, 368–373.
- Rock, I., Linnett, C. M., Grant, P., & Mack, A. (1992). Perception without attention: Results of a new method. *Cognitive Psychology*, *24*, 502–534.
- Trick, L. M., Enns, J. T., & Brodeur, D. A. (1996). Life span changes in visual enumeration: The number discrimination task. *Developmental Psychology*, *32*, 925–932.
- Trick, L. M., & Pylyshyn, Z. W. (1994). Why are small and large numbers enumerated differently? a limited-capacity preattentive stage in vision. *Psychological Review*, *101*, 80–102.
- Vetter, P., Butterworth, B., & Bahrami, B. (2008). Modulating attentional load affects numerosity estimation: Evidence against a pre-attentive subitizing mechanism. *PLoS One*, *3*, 1–6.