Rote versus Rule: Revisiting the Role of Language in Mathematical Thinking

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Abstract

Language is often depicted as the sine qua non of mathematical thinking, a view buttressed by findings of language-of-training effects among bilinguals. These findings, however, have been limited to studies of arithmetic. Nothing is known about the potential influence of language on the ability to learn rules about the relations among variables (e.g., algebra). To test whether arithmetic and algebraic thinking differ, Chinese-English bilinguals were trained to solve arithmetic and algebra problems in either Chinese or English and then tested on new and old problems in both languages. For arithmetic problems, solution times were always longer for English than Chinese; in both languages, solution times dropped during training; after training, solution times continued to drop for old problems, but returned to pre-training levels for new problems. In contrast, for algebra problems, solution times did not differ across language; solution times dropped during training; after training, gains in speed were preserved for both old and new problems. These findings suggest that the contribution of language to mathematical thinking may be limited to the areas of mathematics that are learned by rote and not by rule.

Keywords: language; arithmetic; algebra; mathematical thinking

Introduction

In language acquisition, there is a classic distinction between learning by rote versus learning by rule (Berko, 1958; MacWhinney, 1974; Marcus, Vijayan, Ran, & Vishton, 1999). Rote memorization, for example, is needed to learn the simple past tense of irregular verbs such as “put”, “see” and “go”. These exemplars generalize narrowly (if at all), such that learning the past tense of “put” does not generalize to the past tense of “see”. In contrast, the simple past tense of regular verbs can be learned by rule (e.g., stem+ed) to easily generate “played”, “typed”, and “listened”. Once the rule for regulars is acquired in childhood, the rule generalizes so freely that children often overgeneralize it to irregulars that they had previously been inflecting correctly.

The rote versus rule distinction also seems to play an important role in math learning. Although addition and multiplication could be learned (in principle) by applying Peano axioms, the typical child learns that six times eight equals forty-eight based on rote memorization of the multiplication table. Thus, a child who learns that two times two equals four fails to generalize the axioms that would lead to knowledge that six times eight is forty-eight. In contrast, the knowledge that six times eight equals eight times six is based on the commutative law $a \cdot b = b \cdot a$. And, once learned, the law generalizes to all numbers being added or multiplied.

Although there remains a contentious debate about the reality of abstract rules in language learning (e.g., Marcus, 2003; McClelland & Patterson, 2002; Pinker, 1999), we argue that the rote vs. rule distinction is useful for understanding the role of language in mathematical thinking. Theoretically, this issue is broadly important in cognitive science, arising in such disparate issues as whether thoughts can be separable from the words we use for them, whether the human capacity for mathematics derives from our acquisition of a natural language, and whether some languages enable thoughts that are unique to their native speakers. Practically, it can have educational implications too for the merits of bilingual versus monolingual education.

One prominent view is that language enhances an evolved, approximate sense of number to enable exact mathematical thinking about large numbers (Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999; Spelke & Tsivkin, 2001). In experiments conducted by Spelke and Tsivkin (2001), 8 Russian-English bilinguals were trained on exact and approximate arithmetic problems in each of Russian and English and then tested on both tasks in both languages. The results showed that participants retrieved the answers to exact arithmetic problems faster in the language used during training sessions. In contrast, participants solved approximate arithmetic problems with equal efficiency in both the trained and untrained languages. In other words, they found a language-of-training effect when performing exact as opposed to approximate calculations.

One conclusion that might be drawn from such findings is that natural language is the medium of any mathematical representation (such as large numbers) that transcend the limits of the approximate number system. As Spelke writes, “Human knowledge of number appears to be quintessentially abstract. The concept ‘seven’ appears to transcend any of the particular sets of seven entities that a person enumerates, the particular situations in which she enumerates them, and (one would think) the particular language in which she expresses this enumeration. However, our findings suggest that ‘seven’ is a language-dependent concept, distinct from the Russian ‘sem’, or the French ‘sept’” (Spelke & Tsivkin, 2001, p81). Studies with Amazon indigene tribes that possess only a limited vocabulary for numbers (Frank et al., 2008; Gordon, 2004; Pica, Lemer, Izard, & Dehaene, 2004) and functional imaging studies finding a dissociation between dependence of exact and approximate computations on language-related circuits (Dehaene et al., 1999; Venkatraman, Siong, Chee, & Ansari, 2006) also provide supporting evidence for this view.

However, an alternative interpretation is to consider the role of language in mathematical thinking in terms of rote vs. rule learning. If arithmetic (or the meaning of number words) is learned by rote and stored in associative memory, then any aspect of the learning context (including language of instruction) would be expected to facilitate recall. In contrast, aspects of mathematics that are rule-based and not stored item-by-item in associative memory would be less subject to encoding specificity. On this view, rote-based mathematical thinking, such as arithmetic, involves the memorization of specific number-number associations, generalizes quite narrowly, follows the principle of encoding specificity (Tulving & Thomson, 1973), and thus language dependence. In contrast, rule-based mathematics, such as learning relations among algebraic variables, permits free generalization and is thus language independent.

The Current Study

In this paper, we tested our alternative interpretation by performing a near replication of Spelke and Tsivkin (2001)'s study. Specifically, we trained Chinese-English bilinguals (native language Chinese) to solve a series of arithmetic and algebra problems over a two-day period either in English or Chinese and then tested them on both trained and novel arithmetic and algebra problems in both of the two languages on a third day. If only rote-based mathematical thinking were language dependent, participants would be expected to solve arithmetic problems faster in Chinese than English, and solution times would be expected to drop over training period, but return to pre-training levels for novel problems in the testing period. In contrast, for algebra problems, participants would be expected to solve them with equal speed in both languages, and solution times would be expected to drop over training, remain fast during the testing period, and be equally fast for the old and new problems.

Further, to verify that what was learned in solving algebra problems were rules – i.e., relations among variables – (e.g., $\cdot + \cdot$) and not rote sequences of symbols, answers contained a mixture of foils that either preserved or violated the sequence of relations. In just this case, *rule-violating* foils (e.g., A·B+A+C or A+B+A·C) would be expected to be rejected faster than *rule-preserving* foils (e.g., A·B+B·C).

Methods

Participants

Participants were 40 Chinese-English bilinguals (26 females), ranging in age from 18 to 35 years (M=23.04 years, SD=3.83 years). Participants were recruited from undergraduate and graduate students enrolled at The Ohio State University. All participants were native speakers of Chinese, attended elementary and high school in China, spoke English fluently (but not at native levels), and were comfortable conversing and reading both Chinese and

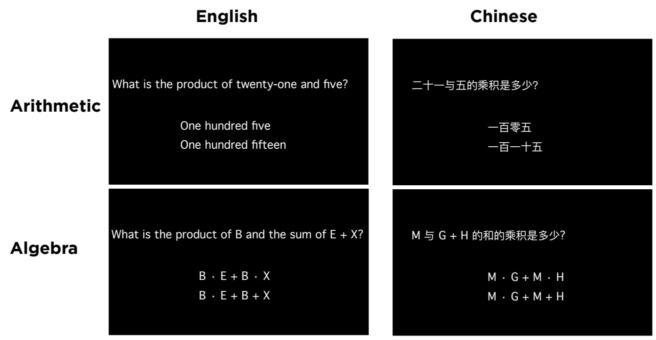

Figure 1: Exemplars of the stimulus display used in different trial types.

English. The university requires applicants with English as a second language to score at least 79 on the TOEFL (iBT), or at least 82 on the MELAB, or at least a 6.5 on the IELTS. These criteria imply English skills sufficient to successfully complete a university degree entirely in English.

Tasks and Materials

The experiment was conducted on a 13-inch MacBook Air laptop, with all the tasks administered using a custom MATLAB program. On each trial of the arithmetic and algebra tasks, one problem was shown in numerical words either in Chinese or English, with two candidate answers presented below (Figure 1). Participants were asked to select the correct answer by pressing the "up arrow" or the "down arrow" key. For the stimuli in both Chinese and English on each task, all questions appeared in Geneva font size 44 and answers appeared in Geneva font size 27. The displays in the two languages on each were designed to be as similar as possible in size and layout.

For arithmetic task, 12 exact multiplication problems were presented, with the first factor ranging from 12 to 28, the second factor ranging from 3 to 9, and the candidate answers ranging from 62 to 146. The alternative answer to the correct one was a number that was 10 larger or smaller. All the problems and answers were the same in the two languages.

For algebra task, 12 algebraic multiplication problems were presented, with the first factor a letter (e.g., A), the second factor a sum of two letters (e.g., B+C). The correct answer was the one following the distributive law (e.g., A·B+A·C). And the alternative answer was either the one with the rule-preserving operands (e.g., A·B+B·C) or with the rule-violating operands (e.g., A·B+A+C or A+B+A·C). To create unique sets of items, different letters were used in English and Chinese versions of the algebra task.

Design

The design was similar to that used by Spelke and Tsivkin (2001). Each participant was given a two-day training session and a one-day test session. In the training session, participants were randomly divided into two groups with one group performing the arithmetic task in Chinese and the algebraic task in English and the second group performing the

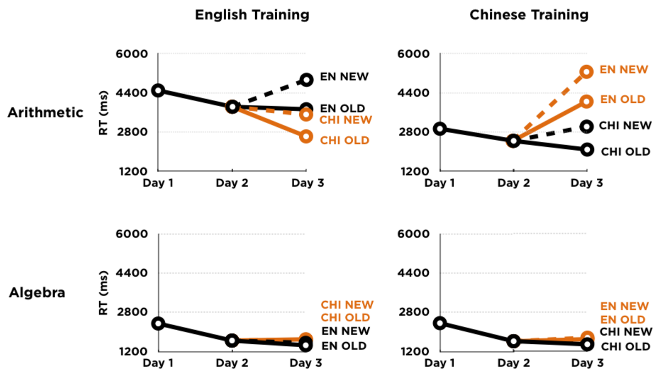

Figure 2: Response latencies on each type of problem and in each language over the 3-day period. Dashed lines=new; solid lines=old; black=trained language; orange=untrained language.

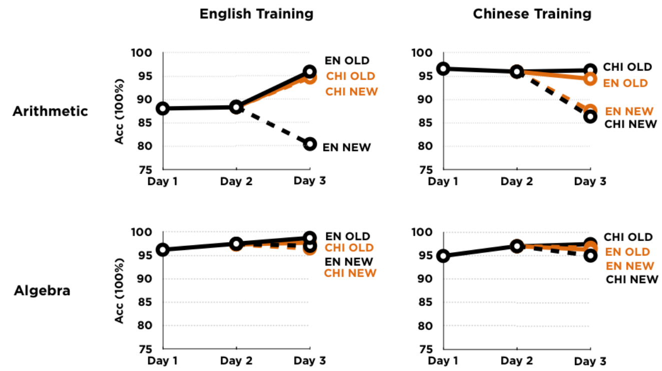

Figure 3: Accuracy on each type of problem and in each language over the 3-day period. Dashed lines=new; solid lines=old; black=trained language; orange=untrained language.

arithmetic task in English and the algebraic task in Chinese. In the test session, all participants were given both tasks and in both languages. The order of languages in both training and test sessions were counterbalanced across participants.

In each training session, each task consisted of 6 repetitions of each of the 12 different problems, for a total of 72 trials per task. In the test session, each task consisted of 6 trained problems and 6 untrained problems. Each of the 12 problems was presented twice, for a total of 24 trials per task in the test session. Problems in each task were presented in a random order and the correct answer appeared on the top and bottom with equal frequency.

Procedure

Before each task, experimenter conversed with the participant in the language to be used for the task. To re-accustom them to working with the corresponding language, immediately prior to each task in English or Chinese, participants were asked to read online news in that language for two minutes.

In the training sessions, each task began with instructions specific to that task and with example problems presented in that trained language. The first trial was showed up by participants pressing the space bar, and remained on the computer screen until the participant pressed a response key indicating whether the top answer or the bottom one was correct. Immediately after that, a feedback specifying whether the response was correct or incorrect appeared on the screen and remained for 600ms. If no response was made within 10s, a third feedback appeared indicating that the trial had timed out. The next trial began immediately after the disappearing of the feedback. Throughout the training sessions, participants were encouraged to respond efficiently, with equal emphasis on speed and accuracy.

The procedure for each task in the test session was identical to that in the training sessions, except that no specific examples were given to explain the task in the instruction.

Results

Results were organized in three sections. In the first section, we examined the training effects for both arithmetic and algebra tasks. In the next section, we examined the performance for arithmetic and algebra tasks in the testing session. In the final section, we examined differences between rule-violating and -preserving operands. Figure 2 and Figure 3 present the mean response latencies and the accuracy for each of the tasks and languages during the 2-day training period and the third-day testing period.

1. Training led to faster and more accurate responses for both arithmetic and algebra tasks, but the effect of language on speed and accuracy was unique to arithmetic.

We first analyzed training session data for arithmetic and algebra tasks. Reaction times (RT) were analyzed for all trials on which a subject gave the correct response within the allowed 10s period. A generalized linear mixed-effect model (GLMM) was conducted, with training day, training language, task, and all interactions as fixed effects, and subjects and trials as random effects. As expected, there were significant main effects of training day, b=-.33, $t(42)$=-9.80, p<.001, task, b=.72, $t(10700)$=62.60, p<.001, and training language, b=-.37, $t(10700)$=-32.38, p<.001. Also, there was a significant interaction of task and training language, b=-.37, $t(38)$=-4.61, p<.001, indicating that participants solved arithmetic problems faster in Chinese than in English (2688.17 vs. 4166.49ms) but solved algebra problems with nearly equal speed in Chinese and English (1992.27 vs. 1988.49ms). An interaction of task and the training day, b=-.03, $t(10700)$=2.73, p<.01, indicated that although training helped lower RT for the second day for both arithmetic and algebra tasks, the decrease was lower for the arithmetic (3680.03ms on day1 vs. 3105.20ms on day2) than algebra task (2347.62ms on day1 vs. 1640.02ms on day2). No other effects were found.

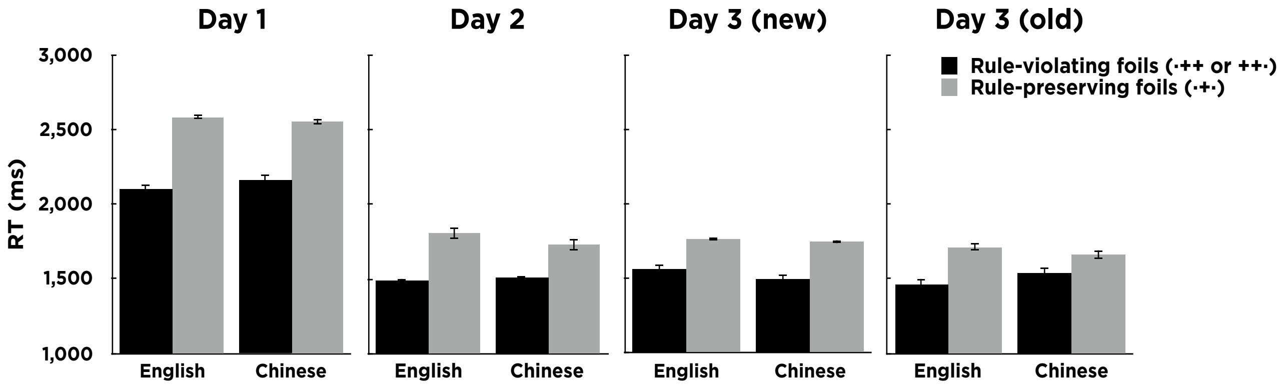

Figure 4: Response latencies of rule-rejecting foils and rule-preserving foils on algebra problems in each language over the 3-day period. Error bars indicate standard error.

Accuracy for arithmetic and algebra tasks were analyzed in a similar way except using a mixed logit model (GLMM for binomially distributed outcomes). There were main effects of training day, b=.17, z=2.72, p<.01, task, b=-.37, z=-8.16, p<.001, and training language, b=.28, z=6.17, p<.001. An interaction of task and training language, b=.35, z=3.48, p<.001, was found, indicating that participants solved arithmetic problems more accurately in Chinese than in English (96.18% vs. 87.50%) but solved algebra problems with nearly equal accuracy in Chinese and English (95.94% vs. 96.70%). An interaction of task and training day, b=-.15, z=-3.35, p<.001, indicated accuracy was higher during the second than the first day of training for the algebra task but not arithmetic task (95.38% on day1 vs. 97.26% on day2 for algebra task; 91.77% on day1 vs. 91.91% on day2 for arithmetic task). Day 1 results are important because they suggest that any pre-existing advantage for the algebra problems were negligible.

2. Dissociation between arithmetic and algebra tasks in the test session.

We next conducted a generalized linear mixed effect model (GLMM) on response latencies for the test problems, with the task, test language, problem novelty, and interactions as fixed effects, and subjects and trials as random effects. Results showed a main effect of task, b=1.02, $t(146)$=19.59, p<.001, language, b=-.42 , $t(146)$=-8.06, p<.001, and problem novelty, b=-.28, $t(24)$=-5.67, p<.001. An interaction between task and language, b=-.41, $t(146)$=-7.96, p<.001, indicated that participants solved arithmetic problems faster in Chinese than English (2799.76 vs. 4419.03ms) but solved algebra problems with nearly equal speed in Chinese and English (1618.03 vs. 1628.19ms). An interaction between task and problem novelty, b=-.25, $t(74)$=-8.50, p<.001, indicated that participants answered old problems more rapidly than new problems for both arithmetic and algebra tasks, but the difference was greater for the arithmetic than algebra task (986.11 vs. 57.89ms). No other effects were found.

Accuracy for the tasks in the test session were analyzed in the similar way except using a mixed logit model. Similar to the results of latencies, results showed a main effect of task, b=-.55, z=-6.88, p<.001, and problem novelty, b=.39, z=2.77, p<.01. An interaction between task and problem novelty, b=.18, z=2.26, p<.05, indicated that participants answered old arithmetic problems more accurately than new ones (95.10% vs. 86.04%), but solved new and old algebra problems with nearly equal accuracy (97.60% vs. 96.36%). No other significant effects were found.

3. Micro-rules were learned through algebra tasks.

Results above suggest that subjects were not just memorizing strings of letters and operations when solving algebra problems. How, then, did they solve the problems? One possibility is that subjects encoded a partial structure of the trained items. For example, subjects might learn a structure of a subset of operations (e.g., ·+·) while solving the algebra problems. If this were the case, rule-violating foils (e.g., A·B+A+C or A+B+A·C) would be expected to rejected faster than the rule-preserving foils (e.g., A·B+B·C) when subjects solving algebra problems.

As Figure 4 shows, participants more quickly solved algebra problems with rule-violating than rule-preserving operands for each day of learning, for both new and old problems, and in both languages. The latency findings were confirmed by a generalized linear mixed effect model (GLMM) on training data and test data.

In the model for the training data, the language (English vs. Chinese), training day (day1 vs. day2), alternative to the correct answer (rule-violating vs. rule-preserving foils), the three two-way interactions, and the one three-way interaction were fixed effects, and subjects and trials were random effects. Results revealed a main effect of alternative choice, b=.18, $t(35)$=6.97, p<.001; a main effect of training day, b=-.36, $t(44)$=-10.68, p<.001; and an interaction of alternative choice and training day, b=-.04, $t(46)$=-2.11, p<.05, indicating the difference on response latencies for answering problems with rule-violating foils versus rule-preserving foils was greater on day1 than day2 (438.81 vs. 260.68ms).

In the model for the test data, the language (English vs. Chinese), problem novelty (old vs. new), alternative to the correct answer (rule-violating foils vs. rule-preserving foils), the three two-way interactions, and the one three-way interaction were fixed effects, and subjects and trials were random effects. Results revealed only a main effect of alternative choice, $b=.10$, $t(25)=3.47$, $p<.01$.

Discussion

Consistent with the rote/rule distinction, our results showed a marked contrast between arithmetic and algebra learning. As in Spelke and Tsivkin (2001)'s experiments, bilingual subjects retrieved answers to rote-based arithmetic problems more quickly in their native language than in their second language, and old arithmetic problems were solved more quickly than new ones. Although we did not replicate the language-of-training advantage found in Spelke and Tsivkin (2001)'s study, full language-of-training effects are typically masked when native languages are too dominant in retrieval (Campbell, 2005; French-Mestre & Vaid, 1993; Kolers, 1968; Marsh & Maki, 1976; McClain & Huang, 1982).

In contrast, our results showed that when the same subjects learned rule-based algebra problems, they retrieved those problems with equal efficiency in their two languages, and this learning could generalize to the novel problems in the testing period. These results suggest that natural language only contributes to rote-based arithmetic learning but not to rule-based algebraic learning.

Our results are broadly consistent with neuropsychological evidence that patients with severe aphasia are able to correctly solve algebraic expressions despite showing impairments in language ability (Klessinger, Szczerbinski, & Varley, 2007), as well as the neuroimaging findings of double neural dissociation of processing algebraic operations and the syntax of language (Monti, Parsons, & Osherson, 2012).

Furthermore, our results showed algebra learning relies on a series of micro-rules, which is the structure of relations among the operations. And these micro-rules can be generalized to new items that do not overlap with the items that appeared in training. The nature of these micro-rules could be a visual perception of the structure, which has been suggested to impact rule application in algebraic reasoning (Landy & Goldstone, 2007; Landy, Jones, & Goldstone, 2008; Marghetis, Landy, & Goldstone, 2016; Schneider, Maruyama, Dehaene, & Sigman, 2012).

We close by discussing three implications provided by this research. First, what might be the role of language in mathematical thinking? Second, what might be the nature of rule-based mathematical thinking? Third, what might be the educational implications of the present findings?

1. Revisiting the role of language in mathematical thinking

Whether natural language does (Bloom, 1994; Carey, 2001, 2009; Lakoff & Nunez, 2000; Nunez & Marghetis, 2014; Paik & Mix, 2003; Spelke & Tsivkin, 2001) or does not (Brannon, 2005; Gelman & Butterworth, 2005) play a role in mathematical thinking is a hotly-debated issue. Although a full review of the subtle differences amongst these views (and their methodological assumptions) is not possible here, we should note that none of these accounts provides a memory systems-based theory predicting a dissociation between arithmetic and algebra (though see Schneider et al., 2012). Moreover, our theory that language effects are limited to rote-based learning generalizes beyond the arithmetic/algebra divide. In addition to arithmetic, verbal-associative learning plays an important role in the acquisition of integer and fraction names, which also seem to exhibit language effects (Frank et al., 2008; Paik & Mix, 2003). Further, in addition to the distributive law, mathematics covers other non-algebraic, rule-governed categories (e.g, parity), where our theory could be tested.

2. The nature of rule-based mathematical thinking

The results of our study suggest that algebra learning does not depend on the particular variables involved but on the structure of relations among the variables. But what is the nature of this representation?

One possibility is that the *spatial* properties of physical layouts are encoded. This was suggested by Landy et al., (2008)'s study, in which they manipulated the visual layouts of algebraic equations and asked subjects to judge whether the equations valid or not. And they found that the physical spacing of algebraic equations impacted subjects' performance, suggesting the role of visual perception in the rule application. In a follow-up study by Schneider et al. (2012), they investigated eye movement during subjects solving arithmetic problems with nested syntactic structure, and found the procedural rules biased the typical left-to-right sequential processing in language. These studies point to the possibility that what is learned in algebra might be language-independent by virtue of a dissociation between spatial and verbal learning mechanisms rather than by a dissociation between rote- and rule-based learning. An interesting test of this hypothesis would be the acquisition of algebraic rules in Chinese by monolingual English-speakers or in English by monolingual Chinese-speakers. In both cases, the spatial layout of the distributive law is preserved, but in the case of monolinguals, the mathematical meaning of the expressions would be unintelligible.

3. Implication in education

The results of our study also have practical implications in education. In many parts of the world, more and more schools provide bilingual education to students. Does bilingual education have more advantages over monolingual education? Our study suggests that the evaluation of monolingual versus bilingual education is complicated and the relative merits will likely depend on whether content must be learned by rote or by rule.

On the one hand, the present study provides evidence that rote-based learning relies on a specific language to encode and retrieve. Therefore, if a child in a bilingual environment is expected to retrieve 'seven times five equals thirty-five' and '七乘以五等于三十五', they will likely need to learn each fact in each language to reach equal facility. As a result, bilingual instruction seems likely to present a heavier burden than monolingual instruction (though obviously this cost may be worthwhile if native language instruction improves understanding). On the other hand, the present study provides evidence that algebra learning shows language independence, suggesting that the choice of bilingual versus monolingual instruction can be made on the basis of considerations other than speed of retrieval.

Acknowledgments

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