Full Day Tutorial on Quantum Theory in Cognitive Modeling

Emmanuel M. Pothos (Emmanuel.pothos.1@city.ac.uk) and James M. Yearsley (James.Yearsley@city.ac.uk)
Department of Psychology, City, University of London, London, EC1V 0HB, UK

Zheng (Joyce) Wang (wang.1243@osu.edu)
School of Communications, Center for Cognitive and Brain Sciences, Ohio State University, Columbus, OH 43210 USA

Peter D. Kvam (kvam.peter@gmail.com) and Jerome R. Busemeyer (jbusemey@indiana.edu)
Department of Psychological and Brain Sciences, Indiana University, Bloomington, IN 47405, USA

Keywords: quantum probability theory; classical/ Bayesian probability theory; Markov processes; contextuality; decision making; memory; similarity.

Introduction

Even though the generally acknowledged normative and descriptive standard for modeling human inference is classical/ Bayesian probability theory (CPT), there have also been several reports which challenge CPT's universal applicability. Some of the most influential empirical demonstrations of such so-called fallacies have been reported by Kahneman, Tversky and their collaborators. For example, consider the evocative conjunction fallacy. In the Tentori et al. (2004) demonstration of the conjunction fallacy, participants are quite happy to consider it more probable to randomly select a Scandinavian person with both blue eyes and blond hair, than just blond hair. Even though we can imagine a line-up of Scandinavian individuals (making the set theoretic structure of CPT explicit and so the impossibility of a conjunction fallacy), there just seems a persistent feeling that somehow the conjunction is more likely than the marginal (cf. Gilboa, 2000). How can our intuition be so much at odds with CPT prescription?

We call quantum probability theory (QPT) the rules for how to assign probabilities to events from quantum mechanics, without any of the physics. QPT is in principle applicable in any situation where there is a need to formalize uncertainty. In psychology, one way to motivate QPT is as a bounded rationality approach to CPT: whereas in CPT we require conjunctions/ disjunctions across all possible questions (and the underlying logical structure is a Boolean algebra), in QPT (classical) conjunctions/ disjunctions are possible only for so-called compatible questions, while for incompatible ones they are undefined (they have to be computed with sequential operations; the underlying logical structure is a partial Boolean algebra).

Where incompatible questions are concerned, QPT provides a radically different perspective on probabilistic inference, compared to CPT, characterized by, for example, interference effects, violations of the law of total probability, supercorrelations, and constructive influences from judgments. These characteristics have provided a rich modeling framework for accommodating behavioral results superficially at odds with classical structure, across several areas including decision making, memory, similarity, perception, and logical reasoning, to mention but a few (overviews in Bruza et al., 2015; Busemeyer & Bruza, 2012; Haven & Khrennikov, 2013; Pothos & Busemeyer, 2013).

The purpose of the tutorial is to provide a comprehensive introduction to the QPT techniques commonly employed in cognitive modeling and illustrate the breadth of cognitive findings for which successful QPT models have been proposed.

Presenters

Emmanuel Pothos is a Professor of Psychology at City, University of London. He has been involved with the quantum cognition research programme since its inception, more than 10 years go. James Yearsley is a mathematical psychologist, originally trained in quantum theory. He has provided one of the most compelling a priori behavioral predictions of QPT (Yearsley & Pothos, 2016). Zheng (Joyce) Wang is a Professor at The Ohio State University. She was Co-Editor for a special issue on quantum cognition that appeared in *Topics in Cognitive Science,* 2013, Vol. 5). Peter Kvam is a postdoctoral researcher at Indiana University, who has published many articles on quantum cognition including in top journals such as PNAS. Finally, Jerome Busemeyer is Distinguished Professor of Cognitive Science at Indiana University and fellow of the Cognitive Science Society. He is one of the instigators of the quantum cognition research programme.

Previous Tutorials and Symposia

The tutorial has been presented at the Cognitive Science meetings in Nashville (2007), Washington DC (2008), Amsterdam (2009), Sopporo (2012), Berlin (2013), Quebec City (2014), Pasadena (2015), Philadelphia (2016), and Madison (2018), with about 30 to 50 participants each time. The ratings from participants after the tutorial were all very positive. In 2017, we held a workshop on quantum cognition supported by the Estes Foundation to 60 participants at a joint meeting of the Society for Mathematical Psychology and the International Conference on Cognitive Modeling at the University of Warwick, UK. Also, this tutorial follows a symposium on quantum cognition at the Cognitive Science meeting 2011, whose papers appeared as a special issue in *Topics in Cognitive Science* (2013).

Assumptions about Participants Background

Most of the techniques we will cover involve elementary linear algebra and should be accessible to participants with minimal mathematical background. Note, no knowledge of physics is required and, with the exception of providing some historical context, no references to physics will be made.

Material to be Covered

We intend to organize the tutorial in three sessions, but with multiple speakers per session and short breaks, to make presentations more engaging for the audience. We note below how each session will be broken up into parts, with an approximate indication of time per part.

Introduction and background (2 hours)

Why employ QPT in cognitive modeling? Busemeyer will provide a brief introduction to the tutorial (0.25 hours). We will then consider a simple QPT model for the conjunction fallacy, explaining how the representations can be set up, how are probabilities computed, and how the interference term necessary to accommodate the conjunction fallacy emerges. We will also discuss the way the QPT prediction of a CF can be interpreted in rational terms (Pothos, 1 hour). We will then provide an overview of empirical findings which have been modeled with QPT, with a focus on other decision findings (e.g., disjunction effect; disjunction fallacy), questionnaire response biases (e.g., order effects), memory (e.g., the overdistribution effect), similarity, and perception (e.g., violations of the law of total probability; Wang, 0.75 hours).

Dynamical models; advanced techniques (2 hours)

We will discuss how dynamical cognitive processes can be modeled with QPT and introduce related technical concepts, e.g., unitary operators and Hamiltonians, side by side with classical counterparts, in the context of well-known empirical results from decision making (Busemeyer, 0.75 hours). We will then introduce some more advanced QPT methods. Notably QPT includes a sophisticated formalism for noise in probabilistic inference (with the formalism of POVMs), that is relevant in psychological processes where noise is assumed to play a substantial role. Additionally, the standard dynamical formalism in QPT can be extended to situations where there is an interaction (information exchange) with the environment (cf. open system dynamics; Yearsley, 0.75 hours). Finally, we will consider Bayesian model comparisons between QPT and matched CPT models and discuss their relative complexity in general terms and in relation to specific examples (Yearsley & Kvam, 0.5 hours).

Generative value (2 hours)

We will consider the generative value of the quantum cognition research programme, with emphasis on explaining the techniques and allowing insight into the thought process leading to model creation. Kvam (1 hour) will present a research programme on modeling heuristics within QPT. In particular, he will demonstrate how several fast and frugal heuristics can be reconstructed by integrating them with a quantum logic structure, introducing qubits, U-gates, and quantum information theory more generally. He will consider several applications including regarding expertise, game theory, and the hindsight bias. Wang (0.75 hours hours) will present one of the most surprising and robust predictions from QPT, the so-called QQ equality, which is a parameter free constraint on how order effects in question pairs ought to add up to zero (Wang et al., 2014). Yearsley (0.25 hours) will discuss the prediction of the Quantum Zeno effect, that the density of intermediate judgments slows down opinion change; this prediction relates to one of the most distinctive properties of QPT, the collapse postulate, which entails state changes from measurements. Pothos (0.25 hours) will illustrate this in a simpler paradigm, leading to a prediction of a novel decision bias. And finally, Busemeyer (0.5 hours) will outline the future directions of the quantum cognition research programme.

Acknowledgments

EMP was supported by ONRG grant N62909-19-1-2000.

References

Bruza, P., Wang, Z., & Busemeyer, J. R. (2015). Quantum cognition: a new theoretical approach to psychology. *Trends in Cognitive Sciences*, 19, 383-393.

Busemeyer, J. R., & Bruza, P. D. (2012). *Quantum models of cognition and decision*. Cambridge, UK: Cambridge University Press.

Gilboa, I. (2000). *Theory of decision under uncertainty*. Cambridge University Press: Cambridge, UK.

Haven, E. and Khrennikov, A. (2013). *Quantum Social Science*. Cambridge University Press: Cambridge, UK.

Pothos, E. M., & Busemeyer, J. R. (2013). Can quantum probability provide a new direction for cognitive modeling? *Behavioral & Brain Sciences*, 36, 255-274.

Tentori, K., Bonini, N., & Osherson, D. (2004). The conjunction fallacy: a misunderstanding about conjunction? *Cognitive Science*, 28, 467-477.

Wang, Z., Solloway, T., Shiffrin, R. M., & Busemeyer, J. R. (2014). Context effects produced by question orders reveal quantum nature of human judgments. *PNAS*, 111, 9431-9436.

Yearsley, J. M. & Pothos, E. M. (2016). Zeno's paradox in decision making. *Proceedings of the Royal Society B*, 283, 20160291.